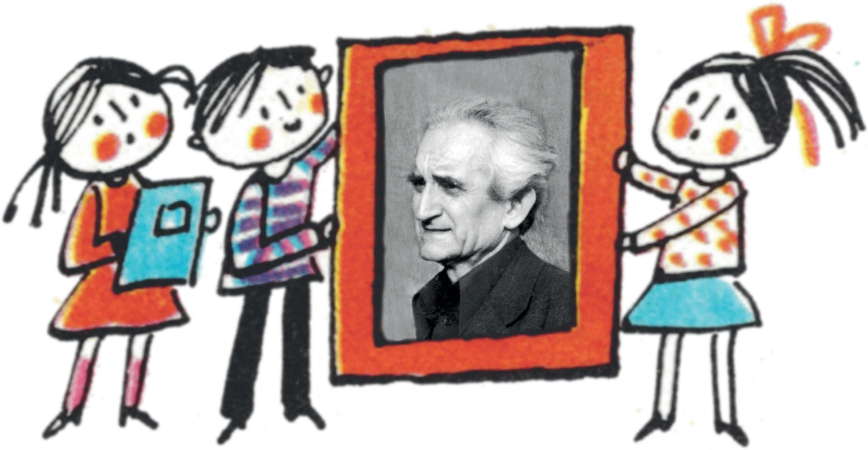

Varga 100

Connecting Tamás Varga's Legacy and Current Research in Mathematics Education

6–8 November, 2019, Budapest

Hungarian Academy of Sciences

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PLENARY LECTURES

Tamás Varga's reform movement and the Hungarian "Guided Discovery" approach

KATALIN GOSZTONYI
Eötvös Loránd University of Budapest
kgosztonyi@caesar.elte.hu
6 November, 9.30 – 10.30

Tamás Varga is one of the major personalities of the history of Hungarian mathematics education. His "Complex Mathematics Education" reform project, led in the 1960s and '70s exerts important influence on Hungarian mathematics curricula, textbooks and teaching practices until today. In the first part of my lecture, I will situate this reform project in its international and national historical context, including the international "New Math" movement and the "Guided Discovery" teaching tradition, something which is embedded in Hungarian mathematical culture. In the second part, I will propose a didactic analysis of Varga's conception on mathematics education. I will especially underline certain characteristics of this conception which can be related to Inquiry Based Mathematics Education.

Reflections on Tamás Varga and an intellectually honest mathematics for all

PAUL ANDREWS
Department of Mathematics and Science Education, Stockholm University (Sweden)
paul.andrews@mnd.su.se
6 November, 11.00 – 12.00

In this talk, having observed Hungarian mathematics teaching for 25 years, I aim to focus on three broad issues relating, directly and indirectly, to Tamás Varga's influence on the teaching and learning of mathematics. The first will be to place his work in context. What was his contribution at home and how did this influence the development of mathematics didactics internationally? The second will be to consider his legacy and examine critically the nature of Hungarian mathematics didactics. In what way is Hungarian mathematics teaching special and what makes it exciting to an international observer? In what way, as the title of my talk implies, does Hungarian mathematics didactics facilitate the creation of an intellectually honest mathematics for all students? Importantly, why has Varga and his legacy been effectively ignored by international scholars? Third, and prompted by the conclusions of the second part of my talk, I will offer evidence showing that

international tests like PISA are poor, possibly inadequate, measures of a country's students' mathematical competence. In other words, should Hungarian mathematics teachers change their practices simply because PISA asserts that Hungarian students' mathematics achievement is poor?

Some logical issues in algorithmic thinking and discrete mathematics

VIVIANE DURAND-GUERRIER
Department of Mathematics, University of Montpellier (France)
viviane.durand-guerrier@univ-montp2.fr
7 November, 16.00 – 17.00

The role of logic in mathematics education has been widely discussed from the seventies and eighties during "modern maths period" till now, and remains still a rather controversial issue in the international community. Nevertheless, the relevance of discrete mathematics and algorithmic thinking as a relevant context for the development of heuristic and logical competencies is one of the main points of the program of Tomas Varga. This relevance has been also underlined for long by the didactic team "Combinatoire naïve et apprentissages mathématiques" in Grenoble (France). In this communication, I will try to show that the didactical issues at the interface of mathematics and computer science raise new questions and new research perspectives on this topic.

Semiotic mediation and cultural artefacts in the mathematics classroom

MARIOLINA BARTOLINI-BUSSI
Department of Education and Humanities, University of Modena and Reggio Emilia (Italy)
mariagiuseppina.bartolini@unimore.it
8 November, 9.00 – 10.00

The name of Varga started to be known by Italian mathematics educators in the 60s of the past century. In particular, Michele Pellerey chaired a group (called RICME = Rinnovamento del Curricolo Matematico Elementare – Innovation in Elementary Mathematics Curriculum) that was strongly inspired by the famous Varga's project at the OPI (Orszagos Pedagogiai Intézet). The influence of RICME on the Italian Elementary School Programs issued in 1985 was very strong and was reconsidered in the Mathematics Curricula prepared later by the Italian Mathematical Union with the names of Matematica 2001 and Matematica 2003. One of the most powerful idea suggested in Matematica 2003 concerns the issue of mathematical

laboratory: A mathematics laboratory is not intended as opposed to a classroom, but rather as a methodology, based on various and structured activities, aimed to the construction of meanings of mathematical objects. A mathematics laboratory activity involves people (students and teachers), structures (classrooms, tools, organisation and management), ideas (projects, didactical planning and experiments). We can imagine the laboratory environment as a Renaissance workshop, in which the apprentices learn by doing, seeing, imitating, communicating with each other, in a word: practicing. This idea, to be implemented, needs tools for analysis and design. This is the prehistory of the theory of semiotic mediation in Italy. I shall illustrate some basic ideas of semiotic mediation, in honour of Tamas Varga, in this centennial.

Many paths lead to statistical inference – should teaching it focus on elementary approaches or reflect this multiplicity?

MANFRED BOROVCNIK
Institute of Statistics, University of Klagenfurt (Austria)
Manfred.Borovcnik@aau.at
8 November, 16.30 – 17.30

The development of methods suitable to tackle the problem of inductive logic – how to justify arguments that generalise findings from data – has been signified by great controversies in the foundations. The controversy was not about the foundations of probability but about the justification of statistical methods: the great players have been on the one side defenders of the classical statistical inference and on the other side the Bayesians. Beyond that controversy there have been several attempts to reconcile the various approaches or to simplify statistical inference. Informal inference subsumes two different areas of didactic endeavour: teaching strategies to simplify the full complexity of inference by analogies, simulations, or visualisations on the one hand, and reduce the complexity of inference by a novel approach of Bootstrap and re-randomisation. All these approaches represent simplifications of the classical statistical inference. There have been only a few approaches to connect statistical inference to decision theory and the Bayesian approach towards inference. One is the parallel approach to classical and Bayesian inference by Vancsó.

The lecture will not go into the details of the controversies but tries to find out what is relevant for teaching inference at high school in the light of this multiplicity. There will also be an attempt to link statistical inference at school to the ideas of Tamás Varga.

PANEL DISCUSSION

Inquiry Based Mathematics Education and the development of learning trajectories

MARIANNA BOSCH[1], MICHIEL DOORMAN[2], PÉTER JUHÁSZ[3], LADISLAV KVASZ[4], KATJA MAASS[5]

Coordinator: MICHÈLE ARTIGUE[6]

[1]IQS School of Management, Universitat Ramon Llull, Barcelona (Spain), *marianna.bosch@iqs.url.edu*

[2]Freudenthal Institute, Utrecht University (Netherland), *m.doorman@uu.nl*

[3]Alfred Rényi Institute of Mathematics, Hungarian Academy of Sciences & Szent István High School, (Hungary), *juhasz.peter@renyi.mta.hu*

[4]Department of Mathematics Education, Pedagogical Faculty, Charles University in Prague & Institute of Philosophy, Czech Academy of Sciences (Czech Republic), *kvasz@fmph.uniba.sk*

[5]International Centre for STEM Education, University of Education, Freiburg (Germany), *maass@ph-freiburg.de*

[6]Laboratoire de Didactique André Revuz, Paris Diderot University (France), *michele.artigue@univ-paris-diderot.fr*

7 November, 9.00 – 11.00

In the last decade, a number of projects have been funded to support the large-scale dissemination of Inquiry Based Education (IBE) in STEM disciplines, in Europe and beyond. However, in mathematics education, well established didactic traditions, among which Tamás Varga's approach in terms of complex mathematics education, share evident values with the current conceptualizations of IBE, and they have supported for decades the development of learning trajectories. In the panel, we will first discuss current conceptualizations of IBME, and clarify their relationship with different didactic traditions. We will then move to the development of learning trajectories within IBME perspectives, once again comparing different approaches and their outcomes. A crucial point is how teachers can be prepared and supported if we want such approaches expand in educational systems. This point will be addressed in the last phase of the panel.

A Film about Tamás Varga

On the occasion of the centenary of Varga's birth, video-interviews were made with Hungarian and foreign colleagues and with family members of Tamás Varga. A 45 minutes long film, based on these interviews, will be presented at the conference on the 7 November, Thursday at 17.00 (during the poster session) in the lecture hall of the Research Centre for Human Sciences.

Exhibition

An exhibition about Varga's person and work will be presented during the conference at its main location, in the Research Centre for Human Sciences.

WORKSHOPS

SESSION 1

6 November, 16.30 – 18.00

Varga's method at the primaryschool level: measurement and unit conversion

ÁGNES KONRÁD
ELTE Gyertyánffy István Primary School, *agnes.konrad@gmail.com*
Room 11

Young learners do not understand the abstract concepts of mathematics such as measurement. In other words, understanding the relationship between the units of measurement and index number is very difficult for them. Thus converting between metric units is also very difficult. Children can only understand these trough concrete, practical experience.

The practical experience such as estimation, measurement by comparison, using standard and non-standard units can support children in understanding.

During this workshop we will discuss how we teach measuring and unit conversion using Tamás Varga's method and complete some suggested activities. We will estimate and compare using non-standard measuring devices to experience how children discover the relationship between the unit and index number.

Varga's method at the lower secondary school level: Tamás Varga, the Wizard, the Explorer, the Game Master

ERIKA JAKUCS
Fazekas Mihály High School, *jematika@gmail.com*
Room 12

In the workshop we collect small morsels from the mathematical roamings of a grade 5 class (students aged 10-11). We get the taste of how deciphering a series of small wizardries leads to a surprise by a mathematical concept that may even turn out to be an old friend. We string some very different problems on a strand (are they that different?), and we may find that the two ends of the strand are connected.

Meanwhile we also savor how grade 5 students explained their ideas, made mistakes, experimented, and got to the goal.

And if someone deciphers the secret of the wizard, they get a chance to enchant the audience themselves.

Varga's method at the high school level: problem solving in elementary geometry

ESZTER VARGA
Bornemisza Péter High School, Eötvös Loránd University of Budapest, *EVarga@bpg.hu*
Room 13

The methods and teaching materials of Tamás Varga's Complex Experiment in Mathematics Education have never been elaborated in such depths for upper secondary education as they were for primary school. Nevertheless, the principles and attitudes of the guided discovery approach are very much present in the practice of some Hungarian expert teachers. As the curriculum gets more formal, the instruction gradually draws away from concrete experiences and manipulatives, but great effort is made to maintain vivid classroom dialog and high responsiveness to students' ideas and contributions. Despite the fact it is time consuming, this teachers sometimes restrain themselves from providing the students with ready-made definitions and methods to give them the opportunity to experience Mathematics as their own creation.

This workshop attempts to simulate this classroom environment. Participants will work on elementary geometry problems in small groups, followed by a whole-group discussion. The problems to be discussed are known in the Hungarian secondary education, but they will be presented in a new arrangement. The workshop will finish with an informal conversation about the experiences and the further potential of the discussed problems.

SESSION 2

7 November, 14.00 – 15.30

A Finnish adaptation of Varga's work: Road to the understanding of the decimal system (primary level)

ANNI LAMPINEN (FINLAND)
Varga–Neményi Association, *anniheinilampinen@icloud.com*
Room 11

Usually we use the decimal system (base ten) to represent numbers. This system is one of the most important concepts in learning mathematics. Understanding the base ten is not an easy task for a young pupil. Varga offers us a versatile approach where pupils can first experience other number bases. When we bear in mind the age of a

pupil the key words are play, joy and doing together! This workshop will give you the experience of learning the decimal system as it is taught in Finland in the footsteps of Varga.

Anni Lampinen is the founder and chairman of the Varga–Neményi Association established in 2005 in Finland. She is also the editor and author of learning and instructional material of the Varga-Neményi method in Finland. She has had a pivotal role in developing the method for the Finnish school context. She was a member of the board which established the new national core curriculum for basic mathematical education 2014 in the Finnish National Agency for Education. She also develops practices and concepts for in-service teacher education.

Teaching mathematically talented students by the Pósa-method

PÉTER JUHÁSZ
Alfred Rényi Institute of Mathematics, Hungarian Academy of Sciences; Szent István High School, *juhasz.peter@renyi.mta.hu*
Room 12

Lajos Pósa is a Hungarian mathematician and educator. Pósa developed a method of teaching mathematics centered on the idea that students should learn to think like mathematicians. Pósa's pedagogy uses the *task thread*, or a series of tasks that build on each other and gradually guide students toward understanding. By engaging with these threads, students discover mathematical concepts through their own work. Initially Pósa's method was intended for talented students and was implemented in more than 350 weekend math camps. Recently a small group has started doing a research implementing it in more general school settings.

The workshop will begin with a brief introduction on Pósa and his work. It follows a short description of basic principles of the method. Then participants will experience Pósa's method by working on several tasks intended for high school students, followed by discussions of the tasks. We will share our experiences of using Pósa's method in Hungarian high school classrooms.

Teacher training for American students in the spirit of Tamás Varga

RÉKA SZÁSZ
Budapest Semesters in Mathematics Education, *reka.szasz@bsmeducation.com*
Room 13

The workshop demonstrates how Budapest Semesters in Mathematics Education—

a study abroad program for American and international preservice and inservice teachers—develops its participants' teacher knowledge. The goal of the program is to introduce participants to the Hungarian mathematics pedagogy through guided discovery, which stems from the work of Tamás Varga. BSME participants often play a dual role: first they are exposed to mathematics tasks in the role of a student; then they reflect on the experience and engage in task design from a teacher's point of view. Workshop participants will experience and discuss this method through tasks involving the Logifaces game. The Logifaces game is a recent Hungarian invention, and a great tool to strengthen students' mathematical thinking and teachers' task design skills.

Alternating Path Algorithm with Party Hats

DÁVID SZESZLÉR[1], JÚLIA KORNAI[2]
[1]Budapest University of Technology and Economics, *szeszler@cs.bme.hu*
[2]ELTE Radnóti Miklós Primary and Secondary School, *julia.kornai@gmail.com*
Lecture hall

A high school graduating class is preparing for their prom. All students in the class give a list of all their classmates of opposite sex they are willing to waltz with. How could we assemble the maximum number of dancing couples? This type of problem comes up in practical, real-life applications too, for example when assigning workers to jobs to be carried out.

Originating from the Hungarian mathematician Dénes Kőnig from the 1930s, the Augmenting Path Algorithm is an efficient method for solving this problem. It has become one of the classics of graph theory, and is often part of introductory discrete mathematics courses at universities around the world.

In this workshop, we present a way to teach this algorithm in an interactive, participatory way: students discover the method and then they perform it together. Finally, the statement and the proof of Hall's Theorem, one of the fundamental theoretical results in this field, is born from the common experience. Obviously, our method follows the long-standing tradition of teaching mathematics in a playful way that focuses on the joyful experience of individual discovery – a tradition that was so successfully promoted and developed by Tamás Varga.

We tested the method twice in summer camps of the ELTE Radnóti Miklós High School, one of the most prominent secondary schools in Budapest. We will also try and share our experiences and observations obtained in these camps.

SATELLITE EVENTS

How can we support students to grasp mathematical structures? Insights into design research in mathematics education – open lecture

SUSANNE PREDIGER
Technische Universität Dortmund, (Germany)
President of the European Society for Research in Mathematics Education

5 November, 17.00 – 19.00

Grasping mathematical structures is a key goal of mathematics education, but has often been shown to challenge many students. The talk reports on design research aiming at understanding and overcoming these challenges. It will show that language is a key learning medium for mathematical structures, and provides insights into approaches and backgrounds how students' language can be supported. The topic serves as exemplary case for explaining how mathematics education research can be conducted as design research.

Teaching statistics with CogStat – workshop

5 November, 13.00 – 16.00

CogStat (*www.cogstat.org*) is an automatic data analysis software with optimized output to reveal the information the statistical analyses rely on. The workshop introduces the use of CogStat, the considerations behind its design, and reviews why it is more effective in teaching environment than many other statistical software.

Venue: ELTE Eötvös Loránd University, *Kazinczy utca 23–27*, 1073, Budapest, room 313

The participation is free for of charges for the participants of the Varga 100 conference but a registration on the website of the workshop is necessary. More information: *https://github.com/cogstat/cogstat/wiki/Workshop-2019-November-Budapest*

XXXI. Tamás Varga Days for Teachers (Varga Tamás Módszertani Napok)

8 and 9 November

Research Centre for Human Sciences

The annual conference for teachers will be joined to the international Varga100 research conference. It will be held at the same location as the international

conference, principally in Hungarian, with one English language section. The program contains presentations and workshops, dedicated to primary and secondary school teachers.

ORAL PRESENTATIONS I.

The tools for developing a spatial geometric approach

ELEONÓRA STETTNER[1], GYÖRGY EMESE[2]
[1]Kaposvár University, Hungary, *stettner.eleonora@gmail.com*
[2]Xántus J. Bilingual Secondary School, Hungary, *gemese2@gmail.com*

Tamás Varga writes about the use of tools: "The rational use of tools – the colored bars, the Dienes set, the logical set, the geoboard, and some other tools – is an element of our experiment that is important for all students, but especially for disadvantaged learners" (Varga, 1977). The range of tools that can be used in teaching has grown significantly over the years. This lecture compares spatial geometric modeling kits. Tamás Varga used the possibilities of the Babylon building set available in Hungary in the 1970s, collected space and flat geometry problems for this (Varga, 1973). Similarly, structured kits with significantly more options have been developed later, e.g. ZomeTool and 4D Frame. These tools are regularly used in the programs of the International Experience Workshop. Teachers, schools that have become familiar with the versatile possibilities of these sets, use them often in the optional and regular classes. Our comparison discusses the possibilities and possible limitations of these tools and compares the experiences of the teachers using these tools. We recorded a lesson on video where secondary students worked with the 4D Frame kit. We also report on the analysis of this lesson.

What is a pedagogy for primary mathematics that most teachers look for – Could Teaching for Mathematising be a solution?

MUN YEE LAI[1], VIRGINIA KINNEAR[2], CHUN IP FUNG[3]
[1]University of Technology Sydney, Australia, *MunYee.Lai@uts.edu.au*
[2]Deakin University, Australia, *V.Kinnear@deakin.edu.au*
[3]The Education University of Hong Kong, Hong Kong, *cifung@eduhk.hk*

Mathematics education in Western countries emphasizes children's development of early number sense (McIntosh, Reys & Reys, 1992). In the past few decades, the teaching of computation has been shifted from presenting algorithms to problem solving process, that is from merely memorizing the computation procedures to helping pupils construct knowledge for themselves. It aimed at improving pupils' mathematical conceptual understanding which in turn fosters their mathematics proficiency. However, different international studies on elementary pupil's mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith; 1997; Mullis, Martin, & Foy; 2008) and PISA (OECD, 2004; OECD, 2010; OECD, 2013) indicated that Australian pupils' performance was

unsatisfactory. This regards items within these tests specifically concerned with four operations of whole numbers (see for example, Mullis, Martin & Foy, 2008, p. 67–70). It seems that this change does not suffice to create quality learning. The question arises to what causes the disconnection between mathematics teaching and learning. The presentation aims at providing a learning trajectory building on the framework of Teaching for Mathematising. In this presentation, one example will be provided for illustrating a learning trajectory for supporting mathematical understanding of both conceptual and procedural knowledge through teaching that builds on the framework of Teaching for Mathematising. A corresponding field-test will then be used to illustrate how students learn pragmatically.

How to lose in lottery games

SÁNDOR DOBOS
Fazekas Elementary and Secondary Grammar School Budapest, Hungary, *dobos@fazekas.hu*

A nice problem appeared in the Tournament of Towns competition (1996, Autumn A level) about a mathlotto. They choose 6 numbers out of 36 and the winning ticket is one which does not contain any of the 6 chosen numbers. How many tickets do we need to get a winning ticket? I showed the problem to Lajos Pósa and it became a regular in the problem sets of his camp programme (Club of Badluck). This inverse-lotto idea grabbed me and I started to play with it. Imagine Unlucky Luke, who is the typical looser; his aim is not to have the winning numbers but to avoid them. The topic is quite motivating for students, changing the parameters can lead us to various concepts of discrete mathematics. The purpose of the talk is to give introduction to the topic and overview of the questions and the results which have already arisen. Parts of this talk appeared in Hungarian on the website of PPKE ITK where I gave a lecture in their teacher in-service training. These problems are really nice problems and they have an important educational aspect. The students can understand that sometimes the final result of a question is not as important as the way which leads to it. It is easy to give upper and lower bounds, then make the estimation sharper using different ideas, approaches. The didactical values are great, the problem encourages dispute, it underlines the importance of changing ideas. I did my best to develop a coherent “series of problems” from the initial question, connecting also different mathematical topics.

Group work at high school according to the inquiry method of Tamás Varga – On beginning of a research

ESZTER KOVÁCS-KÓSZÓ[1], JÓZSEF KOSZTOLÁNYI[2]
[1,2]University of Szeged, Hungary,
[1] kkeszter@math.u-szeged.hu, [2] kosztola@math.u-szeged.hu

The aim of my research project is to develop students' logical thinking. For this reason, Hungarian mathematics teachers need to be encouraged to try new methods which induce greater student involvement. Experiments all over the world prove self-instruction or self-verbalizing has high effect size on the learning process. Collaborative and cooperative technics are also effective, which was one of the key elements of Tamás Varga's experiments in high schools. In my research I am using the "sage-n-scribe" structure of Kagan among 14-18 years old students. It has several advantages like fulfilling the essential elements of cooperative learning and creating situations in which students can practice verbalizing their mathematical thoughts. The second most important quality is the relatively easy feasibility for teachers who are struggling with time when they both prepare for the lessons and while realizing them. This structure is also useful to open the gate toward other collaborative and cooperative technics. During my presentation I will speak about the literature on these topics and I will also present the results of my experiments.

Unusual thoughts in mathematics

FRIED, KATALIN
Center of Methodology of Teaching Mathematics, Institute of Mathematics, Faculty of Natural Sciences, Loránd Eötvös University, Budapest, kfried@cs.elte.hu

Tamás Varga was a great advocate of letting children think their own way (Varga, 1980, e.g.). He embraced this idea so much so that he very much relied on it during his experimental research, making it one of the most important ideas of mathematics education in Hungary. This didactic goal is supported by the researches of psychology of learning as well (Herber, Vásárhelyi, 2006); the joy of discovery is the strongest motive and it also plays an important role of intrinsic motivation. This idea seemingly contradicts the strict and rule-based view of mathematics already at hand. But Tamás Varga made a difference between mathematics at hand and the process of making it, and he pointed out the importance of discovering during making. And the joy of discovery has long-term benefits. I am going to present a couple of examples of unusual ideas which have long term benefits. An example is taken from elementary arithmetic, namely the procedures of operations, some of which Tamás Varga mentioned in his works; and another example will be presented from number theory, or more precisely divisibility rules, that lead to a higher level of thinking. I will also mention some possible long-term advantages in connection with these ideas.

Virtual manipulatives in inquiry based approach of 3D open problems by French 5th graders

ECATERINA PACURAR[1], CLAUDE-ALEXANDRE MAGOT[2], RICHARD CABASSUT[3], YOHAN SOLON[4]
[1,2,3,4]CIREL – Trigone, Lille university, LISEC Strasbourg University, France, [1]*ecaterina.pacurar@univ-lille.fr*, [2]*magot@unistra.fr*, [3]*richard.cabassut@unistra.fr*, [4]*yohan.solon@univ-lille.fr*

The aim of this research is to study the appropriation of a 3D environment by learners in an adidactical situation of open problem solving. We try to evaluate the relevance of the virtual 3D environment in the development of students' cognitive and metacognitive abilities. We propose to verify the following research assumptions: the pupil's immersion in the virtual world reinforces the a-didactic character by identification with his avatar, strengthening the a-didactical situation facilitates the transition of resolution strategies from trial-errors to procedural strategies developed from the mathematical knowledge involved, the motivation for open problem solving will increase as part of a simulation in a 3D immersive environment. For the experimentation, we have implanted in May 2019 in different French primary school areas (in North and in South) a problem solving activity related to a 3D cube situation with an empty part in the cube (with different open problems on counting, magnitudes, net of a solid...). In the experimental group each learner works individually with a PC-computer where the virtual environment ANIPPO is implemented that offers learners the opportunity to evolve in an immersive world on the subject of a city and allow children to solve open problems. A control group follows the problem solving activity in a traditional environment (without computers). The results of the experimentation will be presented at the conference.

Guided Discovery through Problem Threads in Hungarian Secondary Classrooms

MÁRTA BARBARICS[1], BRIDGET GALATY[2], KIRSTEN MANAHAN[3], CHARLOTTE MATZAL[4], HAOYI WANG[5]
[1]Budapest Semesters in Mathematics Education, Hungary, *marta.barbarics@bsmeducation.com*,
[2]Colorado College, USA, *b˙galaty@coloradocollege.edu*,
[3]San Diego State University, USA, *kirstenmarie˙manahan@yahoo.com*,
[4]Wheaton College (IL), USA, *charlotte.matzal@gmail.com*,
[5]Carleton College, USA, *haoyiw97@gmail.com*

In Hungary, 'guided discovery' refers to instruction in which students learn mathematical concepts through tasks and task sequences that foster mathematical

thinking. The pioneer of this method in Hungary was Tamás Varga, who was mostly focusing on the grade 1-8 curriculum in the 1970s (Gosztonyi, 2016). Another prominent figure is Lajos Pósa, who developed his method for gifted students in the 1980s, and also adapted it to the mainstream curriculum (Győri & Juhász, 2017). Rather than teaching mathematics through thematic blocks, the Pósa Method employs webs of interconnected problem threads (Katona & Szűcs, 2017). Additionally, problems within a thread build on each other, meaning that the solution to one problem is necessary for solving the next or that the solutions represent a common mathematical concept. Different threads are presented simultaneously, so that students work on problems from multiple threads at the same time. This presentation communicates the preliminary findings of research conducted by students in the Budapest Semesters in Mathematics Education program as part of a research course. The design and implementation of problem threads in the Pósa Method, with comparison to the US curriculum, are examined through data collected from US and Hungarian literature, as well as interviews, observations, and questionnaires in secondary mathematics classrooms using the Pósa Method in Budapest, Hungary.

How to teach computational thinking for grade 7 students and above with the Pósa method

ESZTER BÓRA
Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics, Budapest, Hungary; ELTE Eötvös Loránd University, Budapest, Hungary, *eszterbora@gmail.com*

Lajos Pósa has been developing his "learning through discover" (Győri & Juhász, 2018) method since 1988. His weekend math camps are focused on fostering problem-solving skills and high-level mathematical-thinking skills in gifted students from grade 7 to 11. One of the core aspects of the method is the structure of the problems, all problems are part of a complex, intertwined and rich network. Through the topic of computational thinking it is going to be shown how this network is structured with the help of the theoretical concept of problem-threads (Gosztonyi, 2019). The success of the method can be attributed in part to the great didactical potential each problem has. With the help of the theoretical framework of didactical situations (Warfield, 2014) these problem-threads are going to be analyzed. The insights gained using his method can be useful in other contexts as well. The possible adaptation of the method to secondary and high schools is also briefly discussed.

Implementing a problem-based learning method: Teaching heuristic strategies in primary school

EMŐKE BÁRÓ (TOTH)
Orbán Balázs Primary School, Romania, *temoke10@gmail.com*

The reflective teachers constantly search for ways to improve the efficiency of their own work, while asking themselves the question about the main goal that they intend to reach. Undoubtedly, one of the significant problems associated with teaching mathematics is the fallacy of motivation to learn. In the presentation, we will discuss teaching methods and learning problem-solving strategies with regard to cooperative activities, and some other student-centered methods, as a solution to promote student involvement in solving mathematical problems. Pupils were put to solve problems using a special heuristic method, this method being pattern recognition and generalization. The classroom observations were analyzed according to different factors, such as adapting to given lesson plans, teaching difficulties, pupils' activity and mood in the classroom. Our measurements were taken both before and after the lessons, using pre-test and post-test to analyze students' results in different situations. Also, a short attitude-questionnaire was given to students both before and after applying these methods. It is not additional, that the activities we designed positively influenced both the motivation and the level of students' knowledge.

Rational errors in learning fractions among 5th grade students

TÍMEA KARIKA
ELTE Eötvös Loránd University, Faculty of Science, Mathematical Doctoral School, Hungary, *makne.karika.timea@gmail.com*

Our paper focuses on an empirical research in which we map out errors in learning fractions. Errors are often logically consistent and rule based rather than being random. When people face with solving an unfamiliar problem, they usually construct rules or strategies in order to solve it (Van Lehn, 1983). These strategies tend to be systematic, often make "sense" to the people who created them but often lead to incorrect solutions (Ben-Zeev, 1996). These mistakes were named by Ben-Zeev (1996) rational errors. The aim of the research is to show that when learning fractions students produce such errors identified in the literature and that the students who make this kind of mistakes achieve low results in mathematics test. The research was made among 5th grade students.

Visualization in geometry education as a tool for teaching with better understanding

JAN GUNCAGA[1], LÁSZLÓ BUDAI[2], TIBOR KENDERESSY[3]
[1]Comenius University in Bratislava, Faculty of Education, Slovakia, *guncaga@fedu.uniba.sk*
[2]Budapest Economical University, Hungary, *budai0912@gmail.com*
[3]Apor Villmos Catholic College, Vác, Hungary, *kenderessyt@gmail.com*

There exists in primary and secondary geometry education some problems with pupils' and students' space thinking and understanding of geometry notions (see Partová, Marcinek, Žilková, Kopáčová, 2013). Visualization plays in geometry education an important role. According Gutiérrez (2014) visual thinking is needed in any area of mathematics at all levels especially in geometry. Development of their visualization skills can support their spatial imagination. This kind of thinking has many practical applications (for example map reading in geography, observing molecular structure in chemistry, space figures of buildings in architecture and engineering and so on). We would like to present our own suggestions about the potential of including visualization, based on the analysis of the Hungarian National Core Curriculum (NAT, 2013, Hungarian National Core Curriculum, 2012). A short comparison between the potentials offered by the Hungarian and the Slovakian curricula will be discussed (ISVP, 2014). After that, we will present selected school tasks, which show how open source software GeoGebra and other visualization tools can help visualize and explain some geometrical notions. Augmented reality (AR) brings new possibility to develop space imagination. We will present this kind of thinking through GeoGebra 3D AR applications. Our goal is to present practical approaches for teachers, which support mathematics education with better understanding.

Web of problem threads and the kernels: preliminary results of the reverse didactic engineering on the Pósa method for IBME, with the use of ATD tools

DÁNIEL KATONA
Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics, & ELTE Eötvös Loránd University, Budapest, Hungary, *danikatona@gmail.com*

The Pósa method, in line with Tamás Varga's ideas, has been in practise since three decades in extracurricular educational settings for talented students in Hungary. The presentation reports on the research carried out by the author within an on-going doctoral thesis, which aims at reconstructing the theoretical frame behind the Pósa method that was never made explicit, through the process called 'reverse didactic engineering', as well as to contribute to the international discourse on IBME. As

a first step of theorizing, independently of other theoretical frames and research results, the model 'web of problem threads' (WPT) and the concept of 'kernel' were theorized, revealing the unique role of the connections between the problems posed in the Pósa camps. As a second step, the Pósa method is analysed with tools of other established theoretical frames within IBME, primarily that of the 'anthropological theory of the didactic' (ATD). Regarding the 'mathematical and didactic praxeologies' used in the Pósa method, the analysis of their 'logos block' reveals new insights into the role(s) of 'kernels'. A sample of the Pósa WPT and some preliminary results on the 'kernels' as core elements of the 'logos block' of praxeologies are to be presented.

A Nim-like game and a machine that plays it: a learning situation at the interface of mathematics and computer science

PIERRE ESCLAFIT[1], SIMON MODESTE[2], NICOLAS SABY IMAG[3]
[1,2,3]University of Montpellier, CNRS, Montpellier, France, [1]*pierre.esclafit@etu.umontpellier.fr*, [2]*simon.modeste@umontpellier.fr*, [3]*nicolas.saby@umontpellier.fr*

This presentation focuses on the links between mathematics and computer science in education. We will present a didactical study regarding the notion of "reinforcement learning" that we have identified as being related to mathematics and computer science. We set in the context of "Computer Science Unplugged". We studied a learning situation where human players are confronted with a machine made of goblets and balls that "learn to play" a Nim-type combinatorial game. After an a priori analysis (based on the Theory of Didactical Situations), we conducted an experiment to validate our hypotheses. During the presentation, we will focus on the analysis of the didactic variables of the learning situation and their effects on the strategies of the students.

Students' approaches while solving a non-typical mathematical problem

BOŻENA MAJ-TATSIS[1], MARTA PYTLAK[2]
[1,2]University of Rzeszow, Poland, [1]*bmaj@ur.edu.pl*, [2]*mpytlak@ur.edu.pl*

In the paper we present the results of two teaching episodes, which took place in two middle school classes with 13 and 14 year-old students. The students in both classes were asked to solve the same mathematical problem; then a discussion followed, in which they had to justify their solutions. In both cases the students had no prior experience in solving non-typical mathematical problems. Additionally, the students were asked to justify their answers, which is not a common characteristic of a 'typical' mathematics classroom at that level. The problem was chosen from a wider research, in which twenty classes from twenty different schools were analysed. One

of the aims of this research related to the present study, was to analyse the skills that require a deeper understanding of mathematical concepts and properties. Particularly, we aimed to investigate students' different solution methods and justifications during problem solving. The results show considerable differences among the two classes, not only concerning the depth of investigating (which was expected due to the different age groups), but also concerning the relationship between achievement (as assessed by the mathematics teacher) and success in solving the problem. These results demonstrate the need for re-directing mathematics education from a pure algorithmic to a deeper thinking approach.

"Pupils' Concepts of a line"

VLASTA MORAVCOVÁ[1], JANA HROMADOVÁ[2]
[1]Department of Mathematics Education, Faculty of Mathematics and Physics, Charles University, Prague, *morava@karlin.mff.cuni.cz*;
[2]Department of Mathematics Education, Faculty of Mathematics and Physics, Charles University, Prague, *jole@karlin.mff.cuni.cz*

Basic geometric concepts understanding is important for the development of pupils' thinking and geometric imagination, both of which facilitate their progress in mathematics. In our research, we focused on the conceptual understanding of selected geometrical concepts among Czech pupils at the end of the 1st, 2nd and 3rd level of education (ISCED 1, 2 and 3). We assigned three tests that were solved approximately by 1,500 pupils/students of various ages as a part of the framework for researching the understanding of geometrical concepts. Two tasks were related to the infinity of a line, another one was related to the number of points belonging to a segment. The gathered data were subjected to a qualitative analysis. We have found that many pupils have not created an adequate concept of a line at the end of the 1st level of education yet and that the misconception remains to adulthood in some cases. In the presentation, we will talk about the results of our research and give some recommendations for mathematics teaching and pre-service teacher training that could help.

No royal road to science – no slavish road either!

ISTVÁN LÉNÁRT
ELTE TTK, ELTE TÓK, Hungary, *lenarti@caesar.elte.hu*

Motto: Students most value their teacher's ability to convince them about their own abilities. As a pupil of Tamás Varga in my early teens, my ambition has always been to distribute and develop his perspective of the subject of mathematics and the profession of teaching. The present talk is a token of appreciation to his memory

and legacy. He had the most profound effect on my way of thinking in scientific and educational research. The main topics are: Comparative geometry on the plane and on the sphere with the help of the Lénárt sphere; making the bridge between the teaching of geometry and geography; connecting non Euclidean geometry with elementary mechanics; applying the comparative method in the study of number systems. These topics illustrate the central messages of Tamás Varga to his pupils and followers. The teacher should be a partner of his/her students in the pursuit of progress and development. Mathematics is not a cold and austere gallery of great men's sculptures from the distant past, but a living, changeable and changing creation of the human mind about our role and tasks during our recorded time in this world. Varga's approach to his science and profession deserves special merit considering the historical era of his active years when his standpoint required courage from the moral and political point of view.

The Role of Visual Representations in Learning Algebra

EMESE VARGYAS
Johannes Gutenberg University of Mainz, Germany, *vargyas@uni-mainz.de*

The use of visual representations is an evidence-based strategy to help students learn mathematical concepts and solve problems. Numerous studies prove that students who use adequate visual representations are much more likely to correctly solve mathematical problems than students who do not use them. A meaningful representation should satisfy at least two requirements. On the one hand, it has to be in accordance with the definition of the concept which is represented by it. On the other hand, a good visual representation should facilitate an effective and successful problem solving. In this talk, we present some problems that have been discussed with prospective teachers to make them aware of the difficulties caused by the discrepancies between the verbal definition of a mathematical concept and the "corresponding" visual representation of it. A discussion about the appropriateness of the chosen visual representation for solving some equations that contain summations completes the talk.

Adaptive Reasoning and Strategic Competence in Croatian Mathematics Education: The example of Quadratic Function

MATEA GUSIC
Faculty of Teacher Education, University of Zagreb, Croatia, *matea.gusic@ufzg.hr*

Mathematical proficiency is a competency framework which proposes five important mutually interdependent strands for successful mathematics learning: *conceptual understanding, procedural fluency, adaptive reasoning, strategic competence* and

productive disposition. This paper aims to investigate *adaptive reasoning* and *strategic competence* in Croatian mathematics education regarding quadratic functions. Existing literature was used to define indicators of these two strands. To address curriculum appointed requirements a commonly used textbook was analyzed. To gain a deeper students' understanding of the strands a case study in the form of observations and interviews was conducted with three second-grade gymnasium students selected considering their ability: a high, an average and a low achiever. The study revealed that the textbook does not provide many tasks potentially promoting adaptive reasoning or strategic competence, implying that the tasks are mainly procedures or exercises. The adaptive reasoning skills of the average- and low-achieving participants are not developed, while the high-achieving participant demonstrated average skills with the highest indicators of plausible reasoning. The participants' strategic competence is in general underdeveloped, relying mostly on memorization and algebraic approaches. Limits of this pilot study refer to the analysis of just one textbook. The stated limit is to be addressed in the future.

ORAL PRESENTATIONS II.

Pre-service teachers' vocational preparation: the case of post-graduate studies qualifying for teaching mathematics as another subject

BARBARA PIERONKIEWICZ
Pedagogical University of Cracow, Poland, *barbara.pieronkiewicz@up.krakow.pl*

During the last decades, many research reports have revealed that in-service teachers of mathematics, as well as full-time student teachers have deep gaps in their knowledge and lack important didactical skills required for teaching. It is puzzling that the problem of the competencies of the pre- and in-service teachers of mathematics who obtain their qualifications for teaching this subject at teacher post-graduate qualifying studies has gained so little attention thus far, at least in Poland. It should be alarming that nowadays teachers may acquire qualifications to teach subjects distant from their basic discipline, for example: a physical education teacher may become a mathematics teacher, within only two or three semesters. Thus, we have a situation that is as much disturbing as absurd. Every year, either the qualifications to teach one of the most difficult and demanding school subjects such as mathematics are obtained by people who have a poor chance of gaining real competencies required to perform this profession, or perhaps we should verify the legitimacy of the preparation for the mathematics teacher profession by attending difficult and demanding studies in the field of higher mathematics. In my talk, I will present some excerpts from a lesson plan and report one lesson on fraction division

conducted by a kindergarten teacher who decided to attend post-graduate studies in order to become a teacher of mathematics. Acknowledgment This work is supported by the National Science Centre (Poland, grant number 2018/31/N/HS6/03976)

Research on proof skills in geometry of secondary grammar school students specialized in mathematics

ÁKOS GYŐRY[1], ESZTER KÓNYA[2]
[1]Földes Ferenc Secondary School, Hungary, *gyoryakos@gmail.com*
[2]University of Debrecen, Hungary, *eszter.konya@science.unideb.hu*

Perhaps the most difficult area of teaching mathematics is to develop students' proving skills. At the end of their studies, secondary school students have to reach the level of ability to form a precise logical chain to prove some simple problems. As a part of a comprehensive, longitudinal research, we monitored proving ability of secondary grammar school beginners (grade 9) specialized in mathematics through geometric problems, based on the work "Van Hiele Levels In Secondary School Geometry" by Zalman Usiskin. We were curious about how students draw up a geometry diagram related to a task, if they can differentiate between the hypothesis and conclusion of a statement, what their written communication is like, and how they reason in case of simple statements.

Can you beat the system? Betting in sports as context for combining classical and subjectivist ideas in probability

ÖDÖN VANCSÓ[1], ESZTER VARGA[2]
[1,2]Eötvös Loránd University, Hungary, [1]*vancso.odon@gmail.com*, [2]*evarga@bpg.hu*, *vargaesztermail@gmail.com*

The motivational value of the context is clearly demonstrated by students in classroom situations posing the title question. This excellent real-life question triggered the design of a special instructional sequence culminating in the addressed problem, and aiming to combine different approaches to probability. Considering students' usual struggle with the field, this attempt may prove valuable. As in the history of Stochastics, investigating the rationality behind the system of odds requires parallel handling of chances and expected value, and can lead to a deeper understanding of complex mathematical models. Recently, Series of Problems have been in the center of attention of the Complex Mathematics Education Research Project in general, as an important characteristic of the Hungarian Guided Discovery approach. Our design roots in this highly problem based tradition and adopts some of Tamás Varga's ideas in instruction. In our presentation, the details and structure of the designed teaching material will be displayed utilizing the tools of this research.

On the visualization of calculating area

GORDANA STANKOV
Subotica Tech – College of Applied Sciences, Serbia, *sgordonka@yahoo.com*

The first year calculus students at Subotica Tech – College of Applied Sciences have big difficulties in learning how to use definite integral for calculating areas of regions that lie under the graphs of functions or areas between curves. In order to help our students, we develop an approach of teaching formerly mentioned calculus contents based on computer supported learning. In this paper we present this approach of teaching using GeoGebra package. It is based on constructivist learning theory, according to which the students explore their learning environment, conduct their own learning activities and construct their knowledge. We analyze learning processes of two students and present their calculations on paper and their work on the computer. During the learning process the students were able to overcome their learning difficulties comparing each step of their calculations with the result obtained using *GeoGebra* commands. We also analyze their answers to questions about their learning process, we posed during interviews. We can conclude that the process of learning, using definite integral for calculating areas with the use of *GeoGebra* is more "constructivist" method, than without computer and that visualization helps in analyzing properties of functions, contributing towards understanding of the whole process of calculation of areas.

The role of teachers' pedagogical content knowledge in how they use textbooks for teaching word problem solving strategies

CSABA CSÍKOS[1], JUDIT SZITÁNYI[2]
[1,2]ELTE TÓK, Hungary, [1] *csikos.csaba@tok.elte.hu*, [2] *szitanyi.judit@tok.elte.hu*

Our research focuses on pre-service and in-service teachers' pedagogical content knowledge concerning word problems in the period when explicit teaching of the solution mathematical steps of problem usually starts. We used the method of personal, individual interviews. 30 pre-service and in-service teachers were involved. Our research questions revolved around the question in which school grade and in what manner explicit teaching of word problem solving strategies should start.

First we reviewed Eastern European textbooks of Romania, Russia, Slovakia, Croatia and Hungary in order to illustrate the phenomenon in a wider historical-cultural context. We have found several similarities among these books.

Based on a comparative document analysis of several Eastern European textbooks, an interview-protocol has been developed.

The difficulty of solving word problems come from many sources, including the linguistic and reading skill factor.

The Role of Arguing, Proof and Logical Thinking in Students' Mathematical Studies and Beyond

MARCELL HORVÁTH[1], ZOLTÁN MOLNÁR[2]
[1,2]Vas Megyei Szakképzési Centrum Gépipari és Informatikai Szakgimnáziuma, Hungary, [1] *h.marcellandor@gmail.com*, [2] *molnarzoltan3@gmail.com*

Reasoning, arguing and logical thinking are gaining a greater role in our present days. Scientific researches have always been emphasised in Mathematical history. To cope with the modern world, to determine the right direction, to shape innovative thinking, to achieve expertise in cooperation – debates and arguments are essential to take the best final decision. In such situations arguments clash, behind which information – results of various mathematical calculations and proofs – can be found. Mathematics education needs to be able to demonstrate all this from early childhood, should it be in individual or group problem-solving, carrying out research, performing project-work, and to all these – developing digital competence even this way. Later on (especially in secondary education), processing more serious mathematical knowledge can play a major role. Related sciences may be involved: mathematics can help to solve everyday problems – physical, chemical, IT, mechanical that may appear in day-to-day life. Therefore, it is crucial even in mathematics for lifelike problems to be issued, as presenting them can truly demonstrate the usefulness of mathematical inventory.

"A whole new vigor"… How to talk to children about math?

LAURENT MAZLIAK
LPSM, Sorbonne Université, Paris, France, *laurent.mazliak@upmc.fr*

A few weeks after the Liberation of Paris, in November 1944, the great museum of living science "Palais de la Découverte" reopened, and offered the Parisian public a series of conferences presenting the various sciences, including some for children. Paul Montel, one of the most famous French analysts of the moment, was asked to pronounce the one devoted to mathematics entitled "Mathematics and Life". In 1947, it was decided to publish this conference in the form of a luxurious quarto book, illustrated by magnificent drawings of the celebrated illustrator Pierre Collot. In my presentation, I will describe the context of this conference and some of the themes chosen by Montel in order to interest children and show them the importance of discipline. Particularly remarkable is the important emphasis on the calculus of probability, a discipline which, however, did not enjoy a particularly developed esteem at the time on the French mathematical scene. This choice of Montel is thus part of a firmly rooted tradition in France (at least since the middle of the 19th century) where one of the main virtues attributed to the calculation of probabilities, alongside some recognized utility (but limited to a few very specific situations such as the so-called law of errors) is to be a source of small problems both entertaining and providing occasions of brain teasing.

Teaching analysis in high-school

HANA BURIAN
Eötvös Loránd University, Hungary, *hana.burian@t-online.hu*

This presentation deals with teaching analysis at secondary school level in Hungary. Calculus was first introduced in high school at the beginning of the 20th century by Beke Manó and Rátz László (influenced by Felix Klein). Both of them emphasised that the most important parts of teaching the new syllabus are demonstrations and exhibitions of practical applications. New Maths has changed the situation in Hungary as well. Calculus concepts are not only being taught illustratively, bur their exact mathematical definitions, using the epsilon-delta and limits concepts are also made familiar to the students. But what do the students understand from this? Most of the research papers about this topic report on the cognitive difficulties that students have with the above-mentioned concepts. The main question of my study is how well do students understand the connection between function properties, and the derivative at a point, or the derivative function? Can they apply their own understanding to questions that are not typical in the mathematical discussions that usually take place in calculus classes? A test of 12 main questions, most of them divided into sub-questions has been developed to analyse their understanding. The questionnaire was created based on a model of understanding developed for the concepts of differentiation and integration (Weigand et al., 2016). The most interesting results will be presented.

Why it is difficult for teachers to apply the principle of active learning?

ESZTER KÓNYA[1], ZOLTÁN KOVÁCS[2]
[1]University of Debrecen, Hungary, *eszter.konya@science.unideb.hu*
[2]University of Nyíregyháza, Hungary, *kovacs.zoltan@nye.hu*

Among the aims of the Complex Mathematics Teaching Experiment (CMTE) led by Tamás Varga in the sixties and seventies in Hungary, Sándor Klein emphasizes the principle of active learning (Klein, 1980). The roots of this idea go back to Polya. "The best way to learn anything is to discover it by yourself." (Pólya, 1981, p. 103). Varga (1988) evaluated the CMTE and he argued that the "pace was too quick" i.e. new curriculum was introduced too quickly; the teachers need sufficient time to be familiar with new approaches. In our paper we investigate another aspect of active learning: the teacher's cognitive load during lessons where he/she let the students think and solve problems by themselves. In a teaching experiment we investigated four teachers' questioning habits and behaviors in such situations where students explain their ideas related to questions requiring high level cognitive demands, e.g. explaining, classifying, inferring, valuing, and judging. We found that one of the barriers to the active learning method is that teachers' high level cognitive load appears when they react to students' arguing in real time. We believe that teachers' self-reflective habits could help them to build complex cognitive schemes which include various students' answers and reactions besides the content knowledge.

Oral presentations I.
6 November, Wednesday, 14.00-16.00

	LECTURE HALL	ROOM 11	ROOM 12	ROOM 13	ROOM 14
	Manipulatives and visualization in the teaching of geometry **chair: Marta Menghini**	**IBME and learning trajectories, different cultures and approaches** **chair: Michèle Artigue**	**Teaching of discrete mathematics** **chair: Zoltán Kovács**	**IBME, students heuristic and strategic thinking** **chair: Michiel Doorman**	**Development of concepts on the early grades** **chair: Csaba Csíkos**
14.00–14.30	Stettner, Eleonóra–Emese, György: The tools for developing a spatial geometric approach	Lai, Mun Yee–Kinnear, Virginia–Fung, Chun Ip: What is a pedagogy for primary mathematics that most teachers look for – Could Teaching for Mathematising be a solution?	Dobos, Sándor: How to lose in lottery games?	Kovács-Kószó, Eszter–Kosztolányi, József: Group work at high school according to the inquiry method of Tamás Varga – On the beginning of a research	Fried, Katalin: Unusual thoughts in mathematics
14.30–15.00	Richard, Cabassut–Pacurar, Ecaterina–Magot, Claude-Alexandre–Solon, Yohan: Virtual manipulatives in inquiry based approach of 3D open problems by French 5th graders	Barbarics, Márta–Galaty, Bridget–Manahan, Kirsten–Matzal, Charlotte–Wang, Haoyi: Guided discovery in Hungarian Secondary Classrooms	Bóra, Eszter: How to teach computational thinking for grade 7 students and above with the Pósa method	Tóth, Emőke–Orbán, Balázs: Implementing a problem-based learning method: Teaching heuristic strategies in primary school	Karika, Tímea: Rational errors in learning fractions among 5th grade students
15.00–15.30	Guncaga, Jan–Budai, László–Kenderessy, Tibor: Visualization in geometry education as a tool for teaching with better understanding	Katona, Dániel: Web of problem threads and the kernels: preliminary results of the reverse didactic engineering on the Pósa method for IBME, with the use of ATD tools	Modeste, Simon–Esclafit, Pierre–Saby, Nicolas: A Nim-like game and a machine that plays it: a learning situation at the interface of mathematics and computer science	Maj-Tatsis, Bożena–Pytlak, Marta: Students' approaches while solving a non-typical mathematical problem	Moravcová, Vlasta–Hromadová, Jana: Pupils' concepts of a line
15.30–16.00	Lénárt, István: No royal road to science – no slavish road either!		Vargyas, Emese: The Role of Visual Patterns in Learning Algebra	Gusic, Matea: Adaptive Reasoning and Strategic Competence in Croatian Mathematics Education: The example of Quadratic Function	

Oral presentations II.
7 November, Thursday, 11.30-13.00

	LECTURE HALL	ROOM 11	ROOM 12	ROOM 13
	Teachers' mathematical and pedagogical content knowledge **chair: Ewa Swoboda**	**Teaching of logic** **chair: Viviane Durand-Guerrier**	**Approaches to probability** **chair: Manfred Borovcnik**	**Analysis and visual tools** **chair: Miklós Laczkovich**
11.30–12.00	Pieronkiewicz, Barbara: Pre-service teachers' vocational preparation: the case of post-graduate studies qualifying for teaching mathematics as another subject	Győry, Ákos–Kónya, Eszter: Research on proof skills in geometry of secondary grammar school students specialized in mathematics	Varga, Eszter–Vancsó, Ödön: Can you beat the system? The combination of different approaches to probability demonstrated in sports betting	Stankov, Gordana: On the visualization of calculating areas
12.00–12.30	Szitányi, Judit–Csíkos, Csaba: The role of teachers' pedagogical content knowledge in how they use textbooks for teaching word problem solving strategies	Molnár, Zoltán–Horváth, Marcell: The Role of Arguing, Proof and Logical Thinking in Students' Mathematical Studies and Beyond	Mazliak, Laurent "A whole new vigor"… How to talk to children about math?	Burian, Hana: Teaching analysis in high-school
12.30–13.00	Kónya, Eszter–Kovács, Zoltán: Why it is difficult for teachers to apply the principle of active learning?	Zimmermann, Alexander: Mathematical grounding from a syntactical point of view – a sentential-logical approach	Wintsche, Gergely: Probability and statistics in the textbooks from 1980	Sajka, Miroslawa: The notion of function as a movement description tool

Oral presentations III.
8 November, Friday, 10.30-12.30

	LECTURE HALL	ROOM 11	ROOM 12	ROOM 13
	Varga's legacy and textbooks **chair: Marta Menghini**	**IBME and teachers' design capacities** **chair: Marianna Bosch**	**Teaching Probability and Statistics** **chair: Ödön Vancsó**	**Manipulatives in mathematics education** **chair: Mariolina Bartolini-Bussi**
10.30–11.00	Kovács, Zoltán–Kollát, Beatrix: A precursor to Varga's reform: the textbook by Gallai and Péter	Varga, Eszter: "How to be well-connected?" Towards the development of Problem Graphs as design tools for teachers	Medova, Janka–Bulkova, Kristina–Ceretkova, Sona: Identification of crucial skills in solving complex problem in probability within the mathematical contest in teams	Partová, Edita–Katarína Žilková: Development of algorithmic thinking in primary mathematical education using linear patterns
11.00–11.30	Szanyi, Gyöngyi–Jánvári, Zsuzsanna: Is it worth revitalizing Tamás Varga's work? – two heuristic strategies in Tamás Varga's textbooks and teacher's manuals	Luxizi, Zhang: From potential to practical variations in the teaching of functions: contrasting Chinese and French cases in higher secondary schools	Fejes Tóth, Péter: Teaching inferential statistics in Hungarian high school – designing a pilot program	Kiss, Anna: Prime building blocks and their multifarious use in the mathematics classroom
11.30–12.00	Szűcs, Kinga: Arguing and proofing in Varga's German-language work	Pintér, Klára: The 'Word Problem Building' helps teachers create series of word problems for 7-12 year-old students	Krajcsi, Attila–Csapodi, Csaba: Teaching statistics with an automatic analysis software	Cabassut, Richard–Fenech, Antoine: Manipulatives and semiotic tools of Game of Go as playful and creative activity to learn mathematics in early grades in France
12.00–12.30	Tuska, Agnes–Benedek, András: Synthesizing the legacy of Varga and Dienes		Princz, Péter–Vancsó, Ödön: Naïve Bayes Classification Algorithm in Python, for the 9-12th grade	Rybak, Anna: Discovering mathematical knowledge by students as the way to successful math learning

Oral presentations IV.
8 November, Friday, 14.00-16.00

	LECTURE HALL	ROOM 11	ROOM 12	ROOM 13
	Varga's philosophical and cultural background **chair: Ladiszlav Kvasz**	**IBME and teacher education** **chair: József Kosztolányi**	**Teaching of algorithmic thinking** **chair: Attila Krajcsi**	**Playfulness in mathematics education** **chair: Paul Andrews**
14.00–14.30	Szmerka, Gergely: The literary and cultural context of Playing with infinity, the book of Rózsa Péter	Szász, Réka–Juhász, Péter–Kiss, Anna–Matsuura, Ryota: Guided discovery for preservice teachers	Medova, Janka: Relation between computational and combinatorial thinking of undergraduate students of applied informatics	Ambrus, Gabriella–Emese, György: Analysis and development of student (open) thinking with word problems based on real situations
14.30–15.00	De Bock, Dirk: Willy Servais and Tamás Varga. A Belgian-Hungarian perspective on teaching school mathematics	Vanegas, Yuly–Gimenez, Joaquin–Diez-Palomar, Javier: Creative mathematical thinking in elementary teacher training	Kalas, Ivan: Computing with Emil in primary school: Setting out for a systematic approach in developing algorithmic thinking	Torre, Matteo: Abstract games in mathematics high school education
15.00–15.30	Golden, Dániel: Imre Lakatos and Tamás Varga: the Hungarian taste for 'informal mathematics'	Kerekes, Judit–Salopek, Gábor: Freudenthal fantasy on the bus, an American adaptation	Csima, Judit: Teaching algorithmic thinking at college level using interactive methods	Kovács, Péter: The "Medvematek" Project
15.30–16.00	Hoffmann, Miklós: Mathematical deconstruction and its methodical aspects		Fried, Katalin–Fekete, István–Princz, Péter: Better understanding mathematics by algorithmic thinking and computer programming	Alpar, Greg: Open Maths: how resistance can turn into curiosity at the university

Mathematical grounding from a syntactical point of view – a sentential-logical approach

ALEXANDER ZIMMERMANN
Pädagogische Hochschule Burgenland, *alexander.zimmermann@ph-burgenland.at*

The transition from school to university mathematics causes many students special difficulties. One reason, perhaps even the main reason, for this is the fact that with this transition mere counting takes a back seat, while (deductive) proving of theorems comes to the fore. Although mathematical-didactic research has shown increasing interest in mathematical reasoning for some time, there is still no adequate definition of the concept of mathematical justification, with all the corresponding negative scientific and didactic consequences. My lecture presents such a definition within sentential logic. The aim is to delineate on a formal level in a didactically appropriate form that rigorous mathematical argumentation which Euclid presented in his Elements within an axiomatic system and which has since been regarded as a model of mathematical reasoning. On the basis of this definition, I develop a software that can be used to specifically promote the learning of proving and the practice of proof techniques in both school and university mathematics lessons. One of the purposes of this software is to overcome the difficulties mentioned above.

Probability and statistics in the textbooks from 1980

GERGELY WINTSCHE
Eötvös Loránd Tudományegyetem, Hungary, *wintsche@gmail.com*

Approximately 50 years ago Tamas Varga's new curriculum changed the teaching of the so-called subject "Counting and measuring". Probability and statistics were two main new topics in Varga's curriculum. I will focus and follow only the occurrence of the probability and statistics topics in the textbooks in the last 50 years. I have examined many textbook series from grade 3 to 8. My selection cannot be complete, but I tried to cover the whole era. I had an easy job at the beginning because there were only one series of textbooks. I focused on three different aspects of the changes: the interaction among the core curriculum, the textbooks and the teachers and their teaching methods. I created measurable indicators for the curriculum and the textbooks, such as the number and the type of the problems and I got a fairly faithful representation of probability and statistics. As expected, the core curriculum had the greatest impact on textbooks and there emerged more games and gaining experience in the newest textbooks.

The notion of function as a movement description tool

MIROSLAWA SAJKA
Institute of Mathematics, Pedagogical University of Cracow, Poland, *miroslawa.sajka@up.krakow.pl*

This contribution is related to one of the cross-cutting themes of VARGA100 Conference described as "Manipulatives and semiotic tools in the development of mathematical concepts" in the context of the notion of function. The following two introductory research questions are posed: (1) How does a human understand or perceive a function in the context of a description of movement? (2) Is the trajectory of movement a primitive notion, as compared to the function graph? The empirical part of the eye-tracking research took place at the Laboratory of Neurodidactics at the Faculty of Mathematics, Physics and Technical Science at the Pedagogical University of Cracow. The results allow to strengthen the hypothesis that trajectory of movement is a primitive notion, as compared to the function graph even for mathematics pre-service teachers. That hypothesis can be a starting point to consider different approach towards shaping the notion of function in school students.

ORAL PRESENTATIONS III.

A precursor to Varga's reform: the textbook by Gallai and Péter

ZOLTÁN KOVÁCS[1], BEATRIX E. KOLLÁT[2]
[1]University of Nyíregyháza, Hungary, *kovacs.zoltan@nye.hu*
[2]Nyíregyháza Lutheran Kossuth Lajos Grammar School, Hungary, *kollat.beatrix@gmail.com*

Katalin Gosztonyi investigated Rózsa Péter's influence on Tamás Varga. While the author primarily focuses on the famous book Playing with infinity in her research, she also raises the assumption that the textbook written by Tibor Gallai and Rózsa Péter for 9th grades in 1949 also influenced the spirit of Tamás Varga's reform as well. In the background of Varga's reform Gosztonyi demonstrates the existence of a coherent heuristic concept of mathematics teaching. In this paper we justify this heuristic approach to mathematics in the Gallai-Péter textbook, and indirectly the influence of this textbook on Varga. Another feature of Varga's conception is the emphasis on developing cognitive skills. In this context, the proof-related activity is in close connection with heuristics. We also analyzed the textbook from this point of view and compared it to textbooks written under Varga's supervision.

"How to be well-connected?" Towards the development of Problem Graphs as design tools for teachers

Eszter Varga
Eötvös Loránd University, Hungary, *evarga@bpg.hu*, *vargaesztermail@gmail.com*

Observation of teachers' design capacity at work is one target of didactical research worldwide, and fostering this capacity is unarguably a possible turning point in the conveyance of mathematical knowledge. In this contribution, a representational tool, the Problem Graph is introduced to explore certain aspects of teachers' design work. In Hungary, the tradition hallmarked by Tamás Varga is particularly demanding towards teachers as they are supposed to be able to design their long term processes very carefully. Thus, practitioners of the Guided Discovery Approach encapsulate a vast amount of teachers' knowledge both into their short term variations and their long term design, that usually manifest itself in the form of Series of Problems. Problem Graphs attempt to reveal the inner structure of this series, as it will be demonstrated in detail through an extensive example from the field of geometry in this presentation. In pilot experiments, the idea of supporting long time process planning with a graph representation resonated with teachers very well, which gave a new perspective to this research: the implementation of Problem Graphs in teacher education as a possible design tool.

Identification of crucial skills in solving complex problem in probability within the mathematical contest in teams

Kristína Bulková[1], Janka Medová[2], Soňa Čeretková[3]
[2]Department of Mathematics, Faculty on Natural Sciences, Constantine the Philosopher University in Nitra, Slovakia, *jmedova@ukf.sk*

With the use of inquiry based learning in mathematics in school practice, the question about problems which can be useful for analysis of students' competencies in IBL, have arisen. In this paper, the Mathematics B-day contest assignment is introduced as a mean to assess the students' performance in mathematical inquiry. Rubrics with didactical variables were designed as a tool for assessing the students' competencies. The key subtasks related to the level of IBL competencies manifested in final mathematical investigation were identified by the means of the statistical implicative analysis of 15 solutions to the assignment Mathematics B-day 2016: "Nice set of dice about non-transitive dice". The ability to produce an algebraic model of stochastic situation was found to be influencing most the mathematical writing within the final assignment. Surprisingly, the reasoning subtasks were not confirmed as having impact of the competencies manifested in final assignment.

Developing algorithmic thinking in primary mathematical education using linear patterns

EDITA PARTOVÁ, KATARÍNA ŽILKOVÁ

Comenius University in Bratislava, Faculty of Education, Slovakia, *partova@fedu.uniba.sk*, *zilkova@fedu.uniba.sk*

Repeating patterns are frequent activities in preschool and primary mathematics teaching. Patterns are useful for developing the ability to identify relationships and rules by which the pattern has been created and also to apply algorithmic thinking. We developed a set of virtual applications ordered by difficulty, available on www.delmat.info. We are also looking for differences between applying rules to virtual or real objects. Virtual applications are focused on completing repeating patterns (step 2), finding and repairing patterns by adding some missing element, or deleting useless element (step 3). The last type of application is adding the sequence of elements to pattern. We observed some categories in children's solutions with virtual and real object which are the same and some differences too.

Is it worth revitalizing Tamás Varga's work? – two heuristic strategies in Tamás Varga's textbooks and teacher's manuals

GYÖNGYI SZANYI[1], ZSUZSANNA JÁNVÁRI[2]
[1]University of Debrecen, Hungary, *szanyi.gyongyi@science.unideb.hu*
[2]Szerb Antal Secondary Grammar School, Budapest, Hungary, *zsjanvari@gmail.com*

Tamás Varga's Complex Mathematics Teaching programme is an illustrious unit of the Hungarian Mathematics Teaching. One of its main characteristics is the Problem Based Discovery Teaching. Heuristic strategies play particularly important role in problem solving. Schoenfeld felt that one of the primal roles of Mathematics teaching research is how to educate heuristics successfully. "Working backward" and "constructing equation" are two strategies closely connected to each other. They play an important role in the process of moving from arithmetic to algebraic thinking in grade 5 to 8. This process presumes that a problem is being solved with "arithmetic equation" (using i.e. "working backward" strategy) then working with "algebraic equation" (using i.e. "constructing equation" strategy). Algebra as a "generalized arithmetic" is an abstract, a phase of which is a procedural part, which can be connected to arithmetic operations and in this phase the solution method of the equations can be connected to the strategy "working backward". Tamás Varga had a statement, that to the maturation of the abstraction is necessary the permanent moving between the concrete and the abstract information. The main goal of our research is to examine and analyze the appearance and the relationship between "working backward" and "constructing equation" strategies in Tamás Varga's texbooks (grade 5^{th}–8^{th}) and teacher's manuals compared to the relevant research results.

Potential, actual and practical variations for teaching functions: Cases study in China and France

Luxizi Zhang
École Normale Supérieure de Lyon, French Institute of Education, France; East China Normal University, Shanghai, China, *zhangluxizi@outlook.com*

This contribution is based on two major hypotheses: variation is the core of task design, and task design is the core of teachers' work. Taking into account variation in task design has a profound theoretical foundation in China France and Hungary, and developing my PhD with two co-supervisors, in China and France, I wish to seize this opportunity for constructing a analytic model of "teaching mathematics through variation" making profit of this theoretical diversity. This model distinguishes potential variation and practical variation and is based on the process of transforming potential variation into actual variation, and of using practical variation for rethinking potential variation. The design of this model is based both on theoretical networking, and on cases analysis, in France and China. In this contribution, we will focus on a critical aspect in the two cases, from potential to practical variation.

Inferential statistics – Accompanying research for the implementation of a new topic in the Hungarian High School

Péter Fejes Tóth
Szent István University, Hungary, *fejestothpeter@yahoo.com*

In Hungarian secondary education, statistics is currently restricted to descriptive statistics with some minor exceptions. To provide a sound basis for the reform of introducing inferential statistics into Hungarian secondary level curricula, the Hungarian Academy of Sciences established a research group in 2016. This group is intended to investigate ways of introducing the topic of inferential statistics that would fit into the current Hungarian curricula and that would be suitable to be accepted and taught accordingly by the teachers. The task is to investigate international approaches to inferential statistics – didactical suggestions of learning paths as well as curricular settings – and adapt them to the Hungarian situation. On-going work is to design teaching activities that are suitable for that purpose and perform workshops with students and selected teachers (with multiplier function in the educational system) to get their feedback for revising the suggested activities. As of date several activities on dice throwing have been worked out and – in a pilot study – tried out. It is about regular and loaded dice and methods to detect or to decide what applies to a specific die. The activity should also – already in the introductory phase – show the kind of information that is crucial for decisions and its quality that is dependent on sample size. Feedback from the first workshop with

selected able students is promising. Next steps will be to test the revised activities with teachers and then to develop materials before we go to formulate the final details of the suggested curriculum that should be approved by the school authorities.

Prime building blocks and their multifarious use in the mathematics classroom

ANNA KISS
Eötvös Loránd University, Hungary, *annaboxx@gmail.com*

This paper is devoted to the presentation of the manifold opportunities in using a little known but powerful mathematical manipulative, the so-called prime building blocks, originally invented by two close followers of Tamás Varga, to support discovery of various concepts in arithmetics including the Fundamental Theorem of Arithmetics (or as it is widely taught, prime factorisation) in middle school.

The study focuses on a teaching proposal to show how students can learn GCD and LCM with understanding, so it also has to address a controversy between higher mathematics (number theory) and currently used teaching methods which may undermine students' understanding. Thus the study discusses the mathematical and methodological background to understanding different aspects of the concept of prime property and highlights the benefits of using prime blocks in scaffolding students' discovery as well.

Although the proposal was designed to be suitable for Hungarian fifth and sixth graders, mathematical context and indications for the use of the manipulative in primary and high school are given.

Arguing and proofing in Varga's German-language work

KINGA SZŰCS
Friedrich-Schiller-University Jena, Germany, *kinga.szuecs@uni-jena.de*

Varga's work focused on children's autonomous activities as well as on their intrinsic motivation in mathematics classrooms and was designed in the sense of the so-called genetic method (Ambrus & Vancsó, 2017, p. 2). His goal was to enable learners to discover mathematics in various every-day situations and to find appropriate mathematical models (Ambrus & Vancsó, 2017, p. 5). In a playful way, he wanted to convey methods, models and basic concepts of mathematics and especially the rules of logic. However, another main area of mathematics is the way of thinking, the way of getting insights, that means the area of arguing and proving in mathematics. The question is how he placed and communicated – supposedly indirectly – arguments and proofs which are suitable for 1st -to-8 th – grade learners. This question will be answered by analyzing his German-language books, especially Engel, Varga & Walser (1974) and Glaymann & Varga (1975). In the talk, some first results of that analysis will be given.

The 'Word Problem Building' helps teachers create series of word problems for 7-12 year-old students – Creating word problems for 7-12 year-old students using a 'Word Problem Building'

KLÁRA PINTÉR
University of Szeged, Hungary, *pinterk@jgypk.szte.hu*

The Hungarian Complex Mathematics Education reform led by Tamás Varga combined elements of the Hungarian mathematical problem solving tradition and the international New Math movements. While 'Guided Discovery' by series of problems was used formerly with gifted students, it became mainstream in ordinary mathematics classrooms. Our research group focuses on 'Series of Problems' (SoP) which are purposely ordered lists of problems created specifically to achieve one or more learning outcomes. One of our goals is to develop a collection of annotated examples of SoP to help teachers construct short term and long term teaching progressions (Gosztonyi, 2019). In my presentation I will show an example of a series of mathematical word problems. We propose a three-dimensional 'building' of word problems. Along the vertical dimension, the 'first floor' contains arithmetical word problems. The problems on the second and third floor need more steps of conclusion – these problems can be solved by an equation or a system of equations by students in grade 7 or above. Younger students can solve these problems by thinking backwards or by representing the problem by a bar model. We show how manipulative tools could support this abstraction. The other two dimensions of the 'building' are based on the mathematical model and the difficulties of the text of the problem, respectively.

Teaching statistics with the support of an automatic analysis software

ATTILA KRAJCSI[1], CSABA CSAPODI[2]
[1]ELTE Eötvös Loránd University, Faculty of Education and Psychology, Department of Cognitive Psychology, Hungary, *krajcsi.attila@ppk.elte.hu*
[2]ELTE Eötvös Loránd University, Institute of Mathematics, Mathematics Teaching and Education Center, Hungary, *csapodi.csaba@ttk.elte.hu*

Statistical knowledge is getting more and more important in our society, and it is getting more important in mathematical education. We develop an innovative statistical software, CogStat (www.cogstat.org), that analyzes data automatically. Its output is optimized to display details revealing the information the analyses rely on and to reveal information that is important in the evaluation of the data. For example, CogStat includes menus for the tasks not for the statistical procedures, it displays the ordinal information for ordinal variables, the chart axes reflect the measurement level of the variables, the software strictly separates the sample and population properties. To evaluate how the software helps understanding statistics, CogStat was used in introductory statistics courses for psychology BA students,

and CogStat was contrasted with the use of SPSS. Teaching sessions revealed that CogStat can ease many details of statistics learning. The carefully designed output not only helped students to understand important pieces of statistical knowledge, but revealed difficulties in understanding that might be hidden in other cases.

Manipulatives and semiotic tools of Game of Go as playful and creative activity to learn mathematics in early grades in France

RICHARD CABASSUT[1], ANTOINE FENECH[2]
[1]Strasbourg University, France, *richard.cabassut@unistra.fr*
[2]Collège international de Strasbourg

The Strasbourg Go Club is an institution that produces the knowledge of the ways to play the game of Go. French primary school is another institution where the mathematical syllabus is taught. We study the double transposition of the knowledge of game of Go and of the mathematical syllabus in the French primary school. In a French IREM (Research Institute on Mathematics Teaching) a group gathers six primary school teachers, two players from the Strasbourg association of game of Go and one researcher in didactics of mathematics. Once per month the research group meets with the following phases: playing and learning game of Go, reporting about the experiments in the classes and sharing produced resources, reflecting on the experiments and conceiving new experiments to implement before the next meeting. Regarding the game of Go knowledge, the experiments show that it is possible to learn adapted game of Go rules. From the pupils' point of view the experiments show that motivation, pleasure, social behavior are developed through game of Go activities. Regarding the mathematical knowledge many parts of the French syllabus of primary school can be taught through the use of game of Go: number, plane geometry, length and area, algorithmic, reasoning, problem solving. The game of Go brings interesting registers of representation and the change of registers is a good way to understand the concepts and the procedures.

Synthesizing the legacy of Varga and Dienes

ANDRÁS G. BENEDEK[1], AGNES TUSKA[2]
[1]Research Centre for the Humanities, Eötvös Loránd Research Network, Hungary, *benedek.andras@btk.mta.hu*
[2]Department of Mathematics, California State University, Fresno, USA, *agnest@csufresno.edu*

Tamás Varga worked closely with Zoltán P. Dienes on "New Math" in Hungary during the 1960s. They examined, evaluated, and perfected the uses of many manipulatives in instruction linked to Dienes's idea of "multiple embodiment". The essence of this method is to provide internally related experiences of creating and discovering abstract concepts, a procedure what Dienes described as "internalized

action". In conducting the kind of instruction that Dienes and Varga preferred, multiple experience plays a crucial role. They focused on the cognitive process of the learning subject in directed learning situations what embodied tools augment by structuring and organizing the learners' experience. We outline a lesson for teachers on formulating and comparing divisibility rules in various bases with the use of Dienes's Multibase Arithmetic Blocks as an illustration for providing such experience. The lesson incorporates the use of manipulatives and visuals in order to enhance communication and independent thinking through struggle in a "confused situation", and to help ripen the concept of number base and place value in the context of divisibility. We argue that Varga's vision for "new math", that also built on the seminal work of Dienes, remains the best vision for keeping the learning and teaching mathematics "fresh", meaningful, and effective anytime, including our present days.

Integrating elements of data science into high-school teaching: Naïve Bayes classification algorithm and programming in Python

Dr. Ödön Vancsó[1], Péter Princz[2]
[1]ELTE TTK, Hungary, *vancso.odon@gmail.com*
[2]ELTE TTK, Ericsson Hungary R&D, Hungary, *princzp@caesar.elte.hu*

Probability theory and mathematical statistics are traditionally one of the most difficult chapters of mathematics to teach. The authors have prior experience in teaching various topics via computer programming. That means not only running a computer program prepared in advance for the pupils, but instead, writing the working code of the problem as a class activity. The idea is to involve computer programming as a didactic tool in hard-to-teach topics. The presentation is about a newly developed curriculum to teach probability theory and mathematical statistics in secondary school. The intended goal is to implement a naïve Bayes classifier algorithm in Python, and demonstrate the machine learning capabilities of it by applying it to a real-world, curated dataset of edible/poisonous mushrooms. The pupils would implement the algorithm in a playful and interactive way. This incremental development process is aligning well with the spirit of Tamás Varga, who considered computers as modern tools of experimental problem-solving.

Discovering mathematical knowledge by students as the way to successful math learning

Anna Rybak
Centre for Creative Learning of Mathematics, University of Bialystok, Poland, *a.rybak@uwb.edu.pl*

Students in many countries have many problems with learning of mathematics. Many students do not like mathematics. It is a problem for teachers too. It is necessary to answer the question: why does math education cause so much problems? At

the University of Bialystok (Poland) we created the Centre for Creative Learning of Mathematics. It is a place where we try to create for students in different ages atmosphere and circumstances for being active discoverers of mathematics, not only the passive recipients of knowledge from books or teachers. As theoretical background we took ideas of Tamas Varga, ideas of Zofia Krygowska, theory of constructivism, strategy of functional teaching of mathematics and problem solving method. Lessons and workshops for students at our Centre are based on combination of these ideas: participants solve practical or theoretical problems (problem solving method) doing concrete, representative and abstract activities (strategy of functional teaching of mathematics created by Z. Krygowska) and all this leads them to discovering and formulating the knowledge (constructivism). The whole process corresponds very well with ideas of T. Varga: students explore knowledge themselves and think independently, and the subject of math is transformed into a thought-formulating process where students step out from passive acceptance into an active role of creation.

ORAL PRESENTATIONS IV.

The literary and cultural context of Playing with infinity, the book of Rózsa Péter

GERGELY SZMERKA
ELTE TTK, Hungary, *Szmerka.gergely@gmail.com*

Rózsa Péter worked together with the well-known figures of the Hungarian didactics of mathematics in the 20th century. Her book is an interesting and important document. On the one hand it summarizes a picture about mathematics, which we can connect with other didactical books, articles (Kalmár, Rényi, Lakatos) and practice (Tamás Varga), on the other hand it is popular until today, so we can see the impact of it on the present culture of teaching mathematics. Rózsa Péter wrote the book for non-mathematicians, especially for people "of literature, of arts, of the humanities". In my presentation first I will focus on this literary context of the book. The text is written in a novel-like, literary form and is full of cultural references (on the history of mathematics but also on other domains of European culture, especially of literature). Among the references we find Marcell Benedek, the Nyugat and the German romanticism, but these connections have deeper roots in the Hungarian culture, and go back to the early 19th century. We can say also, that Playing with infinity is similar to some books from the field of history and theory of literature (see: the works of Antal Szerb, Spengler). I will enlighten these similarities too. In the second part I would like to examine the text from a stylistical approach, focusing on the narratives. We can read the book as a novel, and on the other hand I think there exists a strong similarity between the structure of stories (tales) and the structure of mathematical problem solving.

Guided discovery for preservice teachers

PÉTER JUHÁSZ[1], ANNA KISS[2], RYOTA MATSUURA[3], RÉKA SZÁSZ[4]
[1,2,4]Budapest Semesters in Mathematics Education, Hungary, [1]*peter.juhasz@bsmeducation.com*, [2]*anna.kiss@bsmeducation.com*, [4]*reka.szasz@bsmeducation.com*
[3]Budapest Semesters in Mathematics Education, USA, *matsuura@bsmeducation.com*

In this talk we present teacher training methods used at Budapest Semesters in Mathematics Education (BSME), a study abroad program for American preservice teachers. The aim of the program is for these preservice teachers to learn about the guided discovery pedagogy used in Hungarian secondary mathematics classrooms, stemming from the work of Tamás Varga. At BSME, we challenge preservice teachers' view of mathematics and provide them the tools to likewise challenge their future students. To this end, BSME participants play the dual role of a student and a teacher. As students, they are exposed to tasks of guided discovery; as teachers, they reflect on this experience, and create and teach their own tasks. In the talk, we illustrate this method of teacher training by presenting examples of guided discovery tasks that are used in BSME classes and were originally developed for secondary school students. We also discuss how BSME participants engage in task design and reflection.

Relation between computational and combinatorial thinking of undergraduate students of applied informatics

JANKA MEDOVÁ
Department of Mathematics, Faculty on Natural Sciences, Constantine the Philosopher University in Nitra, Slovakia, *jmedova@ukf.sk*

Mathematicians use computing in their work regularly. The relation between the coding abilities in combinatorics and ability to solve the enumeration and reasoning problems in combinatorics was quantitatively assessed. The solutions of three sets of problems submitted by 45 undergraduate students were submitted to analyses. The solutions were divided into two groups according to the correctness of the solution of the coding problem. The difference in solving enumeration problems between the two groups was not significant. On the other hand, the difference in reasoning task was significant for all three implicit combinatorial models (distribution, selection, partition) of problems.

Analysis and development of student (open) thinking with word problems based on real situations

GABRIELLA AMBRUS[1], GYÖRGY EMESE[2]
[1]Eötvös L. Science University, Hungary, *ambrus.gabriella@ttk.elte.hu*
[2]Xántus J. Bilingual Secondary School, Hungary, *gemese2@gmail.com*

The issue of solving word problems based on real situations has been investigated for decades, from several aspects, especially in the age group of 10-11, but how do students think in other grades if we take a realistic situation, but reduce the complexity of the word problem? How do they deal with a problem that appears to be a simple task, but is different from traditional school tasks (not direct learning of the curriculum, not only one correct answer, no real answer can be given without considering the real situation...)? Our research group has been examining this question in more detail for years, searching for development possibilities, and testing them among teacher training students and secondary school students within the framework of the Hungarian Academy of Sciences' Subject Pedagogy Research Program. The presentation will focus on the process and results of the development activities in a secondary school.

Willy Servais and Tamás Varga A Belgian-Hungarian perspective on teaching school mathematics

DIRK DE BOCK
KU Leuven, Belgium, *dirk.debock@kuleuven.be*

In 1971 Willy Servais and Tamás Varga published Teaching School Mathematics– A Unesco Source Book, a review of curriculum reforms that were under way in different parts of the world. The book, presenting several modern syllabuses as well as examples of classroom techniques and segments of teacher-student dialogues, provided an often consulted guide to the field of mathematics education. We discuss this book and in this way acquire a unique insight into the modernization efforts of school mathematics during the 1960s and early 1970s. We also identify the emergence of new ideas that will prevail over the next few decades. Besides, the book provides a contemporary 'state-of-the-art' of research in (the psychology of) mathematics education, a relatively new field of inquiry at that time. We take this opportunity to discuss Servais' and Varga's views on modern mathematics teaching, as reflected in this book, and compare these views with those of more radical reformers of the 1960s and 1970s, such as Georges Papy and Frédérique Lenger.

Creative mathematical thinking in elementary teacher training

YULY VANEGAS[1], JOAQUIN GIMÉNEZ[2], JAVIER DÍEZ-PALOMAR[3]
[1,2,3]Universitat de Barcelona, Spain, [1]*ymvanegas@ub.edu*, [2]*quimgimenez@ub.edu*, [3]*jdiezpalomar@ub.edu*

In this presentation our objective is to describe the starting position on creative mathematical potential that future elementary school teachers recognize when they reflect on their own proposals for planning and designing activities for the teaching and learning of school mathematics. It relates the promotion of children's creativity, as it was proposed by Varga's notion of the practice of the teaching of mathematics. The study reported here was carried out with a group of future teachers of primary education who were taking the subject: "Management and Innovation in the Mathematics Classroom". In this subject they must plan and design a didactic sequence based upon of a scientific news in a newspaper. After analysing the data collected, we notice that there is a great number of indicators emerging from the dimensions of "openness", "problematization", "exploration" and "validation", while there is less evidence on the dimensions "connections" and "communication". Our students interpret that contextualizing is necessary but it seems difficult for them to describe intra-mathematical connections.

Computing with Emil in primary school: Setting out for a systematic approach in developing algorithmic thinking

IVAN KALAS
Comenius University in Bratislava, Slovakia, *kalas@fmph.uniba.sk*, *i.kalas@ucl.ac.uk*

After more than three decades of developing software environments and pedagogies for teaching programming in England, Netherlands, Hungary, Slovakia, Poland and elsewhere we decided to fully focus on primary stage, where – like in mathematics – computing needs to be correctly initiated, while thoroughly respecting the developmental appropriateness of the pupils and their learning ecology. Thus, we launched our Computing with Emil project, within which an evidence-based curriculum intervention for Years 3 and 4 learners is being designed around carefully identified computational concepts and their breakdown in progressions of suitable preconcepts (J. Piaget). And, unsurprisingly and repetitively, we keep noticing that properly designed programming curriculum results not only in gaining essential computational competencies, but also in numerous opportunities for the learners to encounter powerful ideas related to mathematics (S. Papert), cultivating the learners' mathematical thinking as well. This potential, however, is often hindered by ill interpretations of how the basic computational concepts should be implemented

in lower primary pupils, a drawback amplified by the absence of corresponding evidence-based pedagogy. We will present our strategy which strives to avoid these unwilling defects and has already been adopted by almost 150 primary schools in several countries.

Abstract games in mathematics high school education

MATTEO TORRE
Liceo Scientifico "L. B. Alberti", Valenza (AL), Italy, *matteo.torre1984@gmail.com*

Abstract games are a surprising resource to develop basic skills in mathematics and logic. In the school year 2018-19 I started a project in a 1st class of High School (Liceo) for 8 hours lessons during which we have analyzed, realized and played the abstract games Pentago, Quarto and Hexapawn. These abstract games have contributed to implement students' mathematics, logic and geometric-visual skills. Students realized their poor version with recycled materials of Pentago, Quarto and Hexapawn, played with these games to improve their geometric-visual skills, or using the geometric concepts of rotation and symmetry to identify a winning strategy, and also using probability, combinatory and logic rules to identify the best move. These abstract games help students not only to look at a problem solution as a whole (a few moves in there), but also to introduce coding and AI in an interdisciplinary way. The future perspective is to implement this educational path, analyzing other abstract games (in particular, the Hexapawn 4x4) and, perhaps, to ask my students to invent one of their own.

Imre Lakatos and Tamás Varga: the Hungarian taste for 'informal mathematics'

DANIEL L. GOLDEN
Institute of Philosophy, Research Centre for the Humanities, Hungary, *golden.daniel@btk.mta.hu*

The paper suggests a comparison between the dialogic form of Lakatos' Proofs and refutations and Varga's classroom dialogues, which might offer an important contribution to the theoretical description of Varga's approach – including examination of possible impacts made on both authors by the so-called informal school of mathematics initiated by distinguished Hungarian mathematicians: Alfréd Rényi, Rózsa Péter, László Kalmár. The presentation shall investigate how much in fact the imaginary classroom of Lakatos and the real-life classrooms of Varga may came close to each other. Whether the philosophical model of heuristics suggested by Lakatos can find its place in a system of actual teaching practices implemented for achieving the learning goals defined by contemporary mathematics education in the very spirit of Varga?

Freudenthal fantasy on the bus, an American adaptation

JUDIT KEREKES, GÁBOR SALOPEK
City University of New York, College of Staten Island, *kerekes.judit@gmail.com*

Introduction: In the sixties two mathematicians, Hans Freudenthal in the Netherlands and Tamás Varga in Hungary, had argued that people learn mathematics by being actively involved and investigating realistic mathematical problems. This means mathematizing the world around us by modeling, schematizing, and structuring phenomena.

Method: The goal is to create a welcoming classroom atmosphere in which play takes the front seat. One such scenario is visiting various (animal) stations at the zoo by bus (illustrated by pictures). Passengers are getting on and off the bus at each station (illustrated by arrows), which is modeled on the open number line.

Conclusion and Discussion: A video of a sample lesson in two different settings will illustrate the effectiveness of the Freudenthal-Varga method, which lives on in today's teaching and learning through the various components of cooperative and active learning, by taking ownership in learning, and learning through student dialogue.

Teaching algorithmic thinking at college level using interactive methods

JUDIT CSIMA
Aquincum Institute of Technology, Budapest, Hungary; Budapest University of Technology and Economics, Budapest, Hungary, *csima@cs.bme.hu*

This presentation is about some of the techniques we use at the Aquincum Institute of Technology (AIT) in the course of Algorithms and Data Structures. AIT is a private institution located in Budapest, Hungary, providing a study abroad program for North American undergraduates mainly majoring in Computer Science and Software Engineering. Students mostly spend one semester at AIT in their sophomore or junior year and since they come from various home institutions with different curriculum and structure their background is really very different. The main goal of this contribution is to show both the challenges we meet at AIT in this course and also some of the working practices and interactive methods we use in class to close the existing gap between the backgrounds of the students to make the course interesting and at the same time doable and useful for all the participants. Some evidence about the efficiency of these methods is also presented.

The "Medvematek" Project

PÉTER KOVÁCS
A Matematika Összeköt Egyesület, Hungary, *kpeter@math.bme.hu*

The Medvematek Project aims to eliminate the prejudice against school subjects that develop logical and solution-based thinking (such as mathematics). We address these issues by a comprehensive program that gets children involved from age 10, and follows their development into their university studies. Through innovative tools we engage teenagers with outside the school activities that build upon positive experiences, thus catching their interest, while at the same time we nurture their core skills and develop a community among them. The Medvematek Project reaches around 15000 students annually at its outdoor problem-solving competitions, a series of maths camps and other innovative community-building events. The effectiveness of this program can be measured in multiple ways. The most direct form is through questionnaires, which show overall satisfaction and raised levels of interest in mathematics. An indirect indicator of our success is the number of returning students, which is growing annually.

Mathematical deconstruction and its methodical aspects

MIKLÓS HOFFMANN
Eszterházy Károly University, Hungary, *hoffmann.miklos@uni-eszterhazy.hu*

Philosophy and mathematics, including the didactical questions of mathematics, have been in strong interrelations through many centuries. Concepts of the main stream of the 20th century philosophy (of science), such as logicism, positivism, formalism, structuralism and even social constructivism consider mathematics as a large, massive building with solid axiomatic foundations. This view has influenced didactical concepts as well – in several countries and for several decades teaching of mathematics has been (and still is) somewhat similar to a construction of this building through numerous school years. In this paper we discuss how the recent concept of Derrida's deconstruction can change this view, and what aspects of thoughts and didactical works of Tamás Varga can be philosophically supported by deconstruction, yielding a less frightening and less oppressive methodical approach for the pupils.

Better understanding mathematics by algorithmic thinking and computer programming

István Fekete[1], Katalin Fried[2], Péter Princz[3]
[1]Modern Complex Mathematics Education Research Group, MTA-ELTE, Hungary, *fekete.istvan@inf.elte.hu*
[2]Mathematics Education and Methodology Center, Institute of Mathematics, ELTE University, Hungary, *kfried@cs.elte.hu*
[3]Ericsson Hungary R&D and PhD School of Mathematics and Mathematics Education ELTE TTK, Hungary, *peter.princz@ericsson.com*

One of the most intense periods of Tamás Varga's work is undoubtedly the years between the beginning of the so called Complex Mathematics Education Experiment in 1963 and the official introduction of the new mathematics curriculum in 1978. Science historian István Gazda writes: "Tamás Varga has changed the culture of learning and teaching mathematics, and pedagogy in general in Hungary at all levels, from kindergarten to university." In our talk we examine the question of how algorithmic thinking and program writing, and computer usage in general, contribute to a deeper understanding of mathematics. Tamás Varga has always been interested in algorithms and programmable computing tools, and always thought of programmable devices as modern tools for experimental problem-solving. We are going to mention some concrete examples from the mathematics and computer science education for each school type in a "round trip" – indeed from kindergarten to university. In doing so, we list several examples of the computer supporting mathematics. We will present the detailed curriculum of an after school study group for children aged between 12 and 18, held by one of the authors. Its topics have already been presented at several educational workshops under the title "Experimental Programming in Mathematics".

Open Maths: how resistance can turn into curiosity at the university

Greg Alpar
Open University of The Netherlands and Radboud Univeristy, The Netherlands, *greg.alpar@ou.nl*

Students arriving to the computer science departments often have a fear of mathematics. This fear originates from their prior experiences with a rigid, performance mathematics. Furthermore, students do not see mathematics as a subject in which they can use their own thinking or as a subject that they can apply freely in other contexts. The Open Maths course, a new, introductory university course of about 50 hours study time based on the "mathematical mindsets" methodology (Boaler, 2015), is designed to change students' views in two important aspects.

First, mathematics should be seen as an open, visual subject in which human creativity plays an important role. Second, students should see themselves in mathematics as potential contributors. Even if they are not mathematicians, students can make use of their unique ways of looking at the world and they can learn and develop. The course has a positive effect on curiosity as well as on self-efficacy. Comparing students views before and after the course, we see big changes. In this talk, I am presenting the great and sometimes surprising experiences during two Open Maths courses that were given to computer science students at two universities in The Netherlands: an in-class, “traditional” variant (Radboud) and a mostly online variant (Open University). References Boaler, J. (2015). Mathematical mindsets: Unleashing students’ potential through creative math, inspiring messages and innovative teaching. John Wiley & Sons.

POSTERS

Potentials and barriers of efficacy in math education for unmotivated students

CSILLA BERECZKY-ZÁMBÓ[1], JANKA SZEIBERT[2]
[1,2]Eötvös Loránd University of Budapest, [1] *csilla95@gmail.com*, [2] *jakacsa@gmail.com*

Dunlosky et al. (2013) listed and examined ten different learning/teaching strategies and only two of them were proved to be effective, practice testing and distributed practice. So we designed an experiment in a socially handicapped vocational school to apply these two strategies. The experiment consisted of writing a short "reminder" at the end of each class, on the topic of the class of that day. In order to achieve distributed learning students were asked to hand in a homework via email by Sunday evenings. This experimental design exceeded the capacity of the class. After the first week, no pupil handed in any homework although the teacher regularly reminded them on Mondays and told them that they can hand in until Monday evening.

The control groups were one other class from the same school and two classes of grade 9 in an elite secondary school, ranked in top 10 in Hungary, learning the same topic. The control groups learned the same concepts in the traditional way.

All four groups wrote the same final test when they finished the topics. The results of the students learning using the testing effect were high above the results of the control group in the same school and they achieved similar results as the control groups in the elite high school. This case study is a good indication that the retrieval effect might be helpful in mathematical environment as well.

Digitalized gamification in fifth grade elementary mathematics at rural school

LÁSZLÓ BUDAI
Budapest Business School, University of Applied Sciences, Hungary, *budai.laszlo@uni-bge.hu*

The efficiency, productivity and successfulness pedagogical methods of the 21th century have to be playfulness. Nearly 10 years ago got into the pedagogic public consciousness merely although they applied the game with big successes in the '70 years already in the competitive sector. In the lecture is presented a case study along this innovation. The topic is: 5th class mathemathic lessons about fraction numbers with methods of gamification supported with digitalized tools. The planning, execution and experiences are presented, concerned the results of an impact assessment from the point system started until the opportunity of the level steps. We look for the answer, that the given topic those competences are at the

students' disposal after processing digitalized gamification with a method, which ones in the standard output requirement systems can be expected him. We compared the students' skills how compatible the skills got with the traditional methods. All these a full congruent-validity examination gets onto a presentation from the aim of responding to. References Prievara, T. (2015): A 21. századi tanár (Neteducatio).

Equivalences in quasi-stationary fields (Alternative ways of teaching vectors)

KÁROLY CSERMÁK
Nagykőrös, Hungary, *csermak.karoly@t-online.hu*

My previous publications have analyzed the process of determining the resulting quasi-stationary vector parameters of force vector space (for example, resulting force vector $\mathbf{F}_R$; resulting torque vector $\mathbf{M}_R$, etc.). This new method sets aside the application of the 'classical' vector-calculation algorithms. Its main point is that it discusses the nD-space bounded vectors – defined by unit vectors –, and calculations with them (which have been generally accepted in our everyday practice, as well as in the general education) from a different aspect. According to this new methodology, all components and the so-called PR 'reducing point' are arbitrarily positioned, relative to a randomly selected 'base vector', regardless where that is located in the unknown space. By the application of this methodology, I will demonstrate how to ensure the stability of the size and/or vector components of a $\mathbf{K}_R$ resulting vector, as well as the consequences of this in the vector component system.

The Pólya-Method in Practice – Short abstract for booklet

EDITH DEBRENTI[1], BALÁZS VÉRTESSY[2]
[1]Partium Christian University, *edit.debrenti@gmail.com*
[2]University of Debrecen, *verstessy01@gmail.com*

The most important steps in solving difficult, problem-type tasks during the lesson are suggested to be done by Pólya's model, which develops problem-solving thinking. The phases of problem-solving have four major steps (Pólya, 2000). The subject of this research is a learning experiment which was done with sixth- and seventh grade pupils. They worked with the Pólya-problem solving model. We built and presented the lesson by using this model. With the help of auxiliary tasks, the class have arrived at the original, more difficult task, which was first transformed, several times, so the class could solve it. We encouraged the pupils to generalize, observe the rule and then we returned to the original task. In the end, we asked them to create a task. The experiment was important for us because we have found just a small number of examples of concrete application of the Pólya-model for lower grades.

Is it possible to develop metacognition in classroom environment?

MÁRTON KISS[1], ESZTER KÓNYA[2]
[1,2]University of Debrecen, Hungary, [1]*kmarci88@gmail.com*, [2]*eszter.konya@science.unideb.hu*

In an earlier exploratory survey we investigated the metacognitive activities of 9th grade students. We have found that they have only limited experience in the "looking back" phase of the problem solving process. As soon as any result is calculated, they immediately stop the work. They don't pay attention to whether the solution is realistic or not; nor do they check the correctness of their results. With the use of these findings, we have elaborated a teaching experiment for an average mathematics class aimed at helping 9th grade students develop metacognitive abilities. During the action research we focused on having the students understand the problem (1), look for more than one solution (2); check the result (3) and review the applied solution method (4). The analysis of the teaching experiment is based on students' pre-, post- and delayed tests, as well as on notes made during and right after the experimental lessons. On the poster we present some tasks and students' solutions from the tests and summarize the experience. We believe that there are many opportunities in the classroom to develop students' metacognitive abilities and thus make their problem solving process more effective.

After Varga: Let's Play Math!

ZOLTÁN KOVÁCS[1], KLAUDIA KAVALECZ[2]
[1]University of Nyíregyháza, Hungary, *kovacs.zoltan@nye.hu*
[2]STEM Experience Center Nyíregyháza, Hungary

Applying the principles of the Complex Mathematics Teaching Experiment (CMTE) led by Tamás Varga in the sixties and seventies in Hungary, we have developed a mathematics program at the STEM Experience Center in Nyíregyháza for 5-12-graders. Some tasks and tools used by Tamás Varga in CMTE were used in the same way, while others were adapted to the modern program. In this paper we focus on "Space and Plane" topic, where Tamás Varga used e.g. the "Babylon" game, the modern form of which is the "Zometool" building tool. We also report on classroom social climate which was investigated on the base of students' drawings of the classroom. We considered students' drawings as external representations of their perceptions of classroom social climate.

Use of Augmented Reality to Develop the Ability to Estimate in STEM Education

LILLA KORENOVA[1], IBOLYA VERESS-BÁGYI[2]
[1]Comenius University in Bratislava, Faculty of Education, Slovakia, *lillakorenova@gmail.com*
[2]University of Debrecen, Doctoral School of Mathematical and Computer Sciences, Hungary, *veressba@gmail.com*

In our contribution we want to point out to the possibilities of using AR (Augmented Reality) in teaching and learning mathematics. We conducted research on the use of AR applications for smartphones which are used to measure length, circumference and area. We proceeded from the principles of constructionist teaching/learning and STEAM (Science, Technology, Engineering and Mathematics) education principles. The study is also based on the European Framework of Key Competencies. One of the main tasks of school mathematics is to create a connection between school mathematics and the real world. Connecting mathematics to real life is one of the reasons students learn to estimate in mathematics. We focused on the possibilities of using available AR applications in order to develop the learners' ability to estimate measurable quantities (length, area, volume, angle, etc.). The research was conducted with the participation of pre-service teachers (students of teaching) at The Comenius University in Bratislava (Slovakia) and the University in Ostrava (Czech Republic). The research design is a combination of methods–qualitative (observation) and quantitative (questionnaire). Via the questionnaire, we researched the views and attitudes of students on this kind of teaching.

Teaching algebraic expressions using the history of mathematics

ZOLTÁN MATOS
Elementary and Grammar School of University of Szeged, Hungary, *matos@freemail.hu*

Algebraic expressions are challenging in secondary school for teachers and students of grade 9 as well. At the beginning of their algebra study most of the students find the topic too abstract and rather useless. The teacher, who is aware of the importance of algebraic expressions regarding to other mathematics topics (equations, functions, coordinate geometry etc.), has only a few opportunities to give students interesting exercises for practice. The integration of historical aspects into the compulsory curriculum helps to better understand the algebraic notation and makes lessons more diverse where students solve routine tasks. On the poster I present particular lesson plans where the historical content appear.

Analysis for Non-Mathematician Students, A Student-friendly Calculus

MUNKÁCSY KATALIN[1], STETTNER ELEONÓRA[2]
[1]ELTE, Hungary, *katalin.munkacsy@gmail.com*
[2]Kaposvár University, Hungary, *Stettner.Eleonora@ke.hu*

We use Tamás Varga's mathematics teaching concept to teach analysis. We use elements from the history of teaching analysis and the results of contemporary research on teaching analysis. According to László Lovász, the concepts of continuity and limit are too difficult for students who are not sufficiently mature in mathematics, so it is worth delaying the teaching of analysis as much as possible. However, in most undergraduate courses, first-year undergraduates need to know and apply calculus (also known as introductory analysis, i.e. derivation, integration). The advancement of mathematics didactics does make it possible to resolve this contradiction. Of these results, we attach particular importance to the appreciation of the role played by games and the increasing use of computers. Computer-generated animations allow us to monitor changes and experiment actively. In this poster, we would like to present the conceptual framework of the calculus we propose and give examples of tried-and-tested game-like tasks and computer applications. The first phase of an exploratory research process is completed. Experimental testing of our research is in progress, and we will be able to present some elements of these experiences at the conference.

Using real and unreal artefacts in developing algebraic thinking

IZABELA SOLARZ
Pedagogical University of Cracow, Poland, *izasolarz@interia.pl*

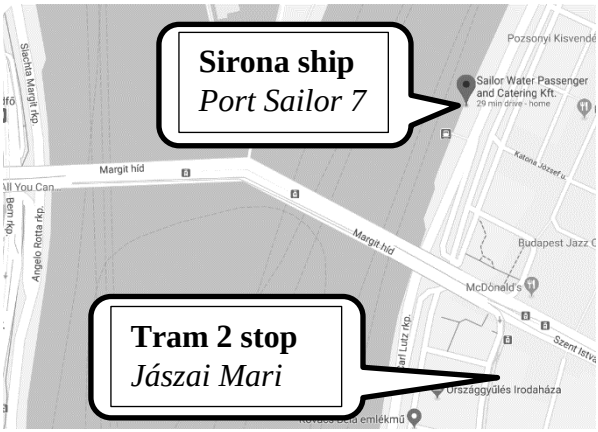
The aim of the classroom experiment presented in this contribution was to answer the question: How the use of computer games and special blocks, can foster the process of learning – teaching early algebra? I run the experiment with a group of twenty 12-years old children, who used three different artefacts during mathematics lessons. Choosing every tool, I considered different difficulties that students encounter – especially when solving linear equations. Using the game was connected with troubles with performing operations on both sides of equations and with the sign of equality. The blocks were used for better understanding of the concept of solution and the concept of variable. The "cover – up" application gave opportunity to get used to the construction of algebraic expressions and connections between opposite operations. The results showed what obstacles children could overcome.

PRACTICAL INFORMATION

Gala dinner

7 November 19.30 – 22.30

The gala dinner will take place on a sightseeing ship on Thursday, 7th November, beginning at 19.30. The ship Sirona will start off from port 7, about 150 meters north of the Pest side of Margaret Bridge. We go to the ship together from the venue of the conference, leaving at 18.45. Attention! Due to the nature of the event, those arriving late will not be able to attend the dinner, please pay extra attention to be in time. The event will end at 22.30. Those who paid for the gala dinner will find a ticket in their badge holder to take with them to the dinner. In case of any problem, please contact the organising team.



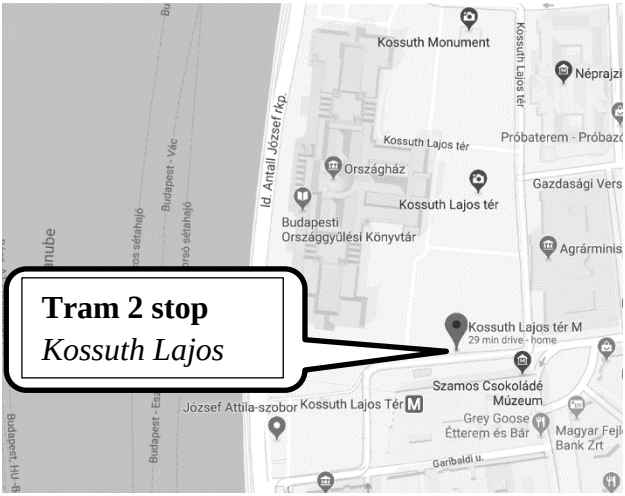
Guided tour

9 November 10.00 – 13.00

The guided tour entitled *Golden Age Budapest – The glittering years of the Austro-Hungarian Monarchy* will start on Saturday, 9 November, at 10am. The meeting point is at Kossuth Lajos Square at the stop of the tram line 2, at the exit of the underground station. The tour is organized by Hosszúlépés (https://hosszulepes.org/en).

Description of the tour

We start our tour by telling the story of the unification of the three cities, Buda, Pest and Old Buda on the Danube shore, while enjoying unparalleled view to the Chain Bridge and the Buda Castle. We visit Saint Stephen's Cathedral, one of the most well-known landmarks of the city, then explore the history of downtown Budapest. We visit a market hall, take a peek in some of the most fabulous, closed courtyards of the city then finish our tour in the heart of Budapest, in front of the State Opera House. Those who paid for the guided tour will find a ticket in their badge holder to take with them to the tour. In case of any problem, please contact the organising team.



Parking

At the conference venues we cannot provide free parking for you. If you travel in Budapest by car, please do not forget that in most parts of the city you have to pay parking fees with the help of vending machines on the streets.

WiFi

Internet will be available at the conference venues, please look for the indications of the network name and the password.

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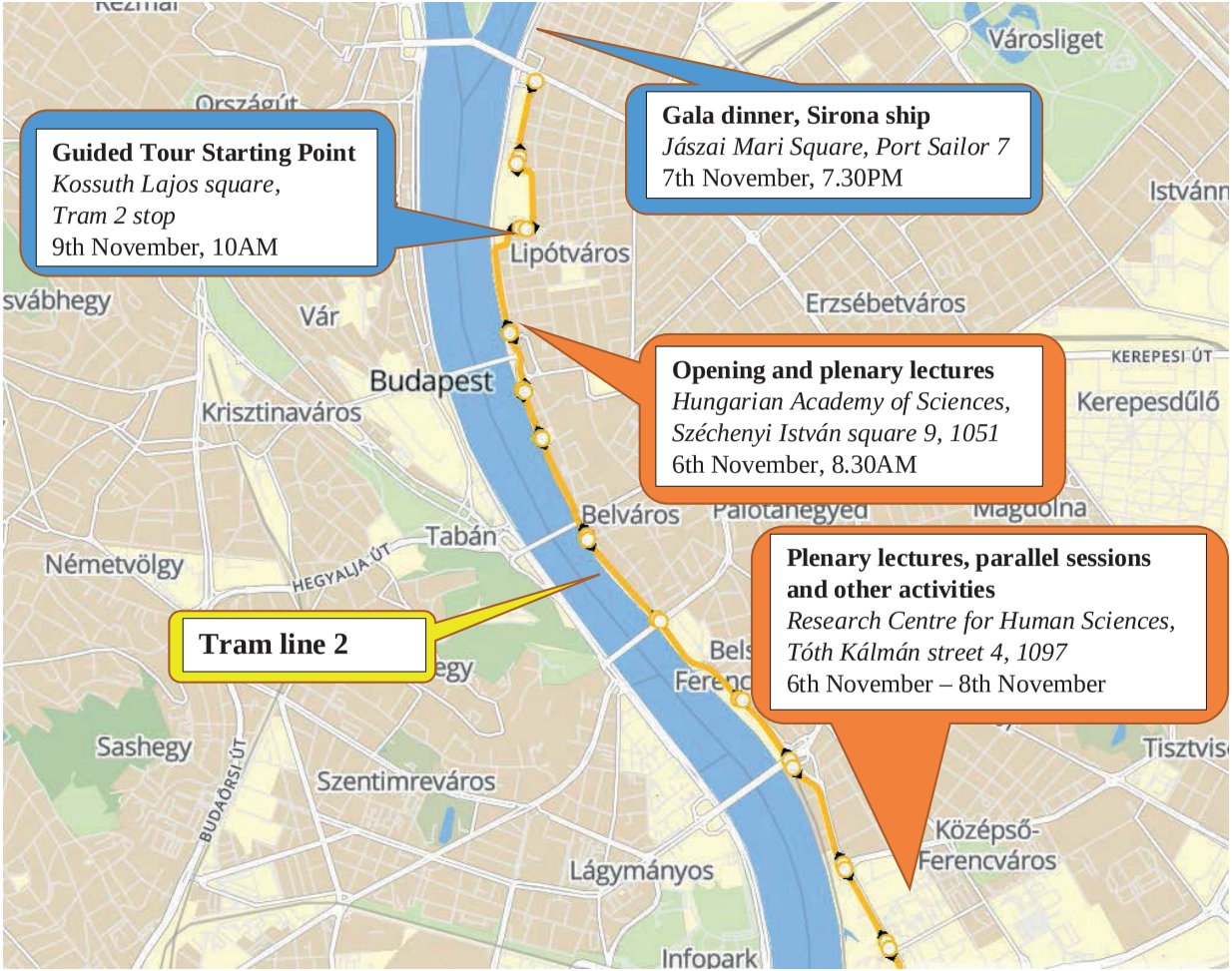
Városliget
Országút
Guided Tour Starting Point
Kossuth Lajos square,
Tram 2 stop
9th November, 10AM
Gala dinner, Sirona ship
Jászai Mari Square, Port Sailor 7
7th November, 7.30PM
István
Lipótváros
svábhegy
Vár
Erzsébetváros
KEREPESI ÚT
Budapest
Kerepesdűlő
Krisztinaváros
Opening and plenary lectures
Hungarian Academy of Sciences,
Széchenyi István square 9, 1051
6th November, 8.30AM
Belváros
Palotanegyed
Magdolna
Tabán
Németvölgy
HEGYALJA ÚT
Plenary lectures, parallel sessions
and other activities
Research Centre for Human Sciences,
Tóth Kálmán street 4, 1097
6th November – 8th November
Tram line 2
Sashegy
BUDAÖRSI ÚT
Szentimreváros
Középső-
Ferencváros
Lágymányos
Infopark

	Tuesday 5 November 2019	**Wednesday 6 November 2019**	**Thursday 7 November 2019**	**Friday 8 November 2019**	Saturday 9 November 2019	
8.30		Registration				
9.00		Opening	Panel discussion	Plenary lecture M. Bartolini-Bussi		Satellite event: Tamás Varga Teachers' Day
9.30		Plenary lecture K. Gosztonyi				
10.00				Coffee break	Guided tour	
10.30		Break		Parallel sessions (Oral presentations III.)		
11.00		Plenary lecture P. Andrews	Coffee break			
11.30			Parallel sessions (Oral presentations II.)			
12.00		Change location				
12.30		Lunch		Lunch		
13.00	Satellite event: CogStat workshop		Lunch			
13.30						
14.00		Parallel sessions (Oral presentations I.)	Workshops	Parallel sessions (Oral presentations IV.)		
14.30						
15.00						
15.30			Coffee break			
16.00		Coffee break	Plenary lecture V. Durand-Guerrier	Coffee break		
16.30		Workshops		Plenary lecture M. Borovcnik		
17.00	Satellite event: Susanne Prediger open lecture		Poster sessions; Film about Varga			
17.30				Closing		
18.00						
18.30				Satellite event: Tamás Varga Teachers' Day		
19.00						
19.30			Gala diner			
20.00						